Multiscale perturbative approach to SU(2)-Higgs classical dynamics

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In collaboration with

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April 7, 2012

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Introduction

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- From the classical point of view:This treatment is insufficient,because such a perturbation scheme develops instabilities due to the emergence of secular terms at higher orders.
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- In our approach, the classical equation of the SU(2)-Higgs model are solved approximately, using multiple-scale perturbation theory to handle the nonlinear terms.

- The multiple scale technique employs slow temporal and spatial scales to reduce the original nonlinear system to a simpler one.

$$f(x^\mu) \to f(x^\mu, x_1^\mu, x_2^\mu, \ldots)$$

$$x_i^\mu = \epsilon^i x^\mu, \qquad \epsilon \ll 1$$

Then :

$$\frac{\partial}{\partial x^\mu} = \sum_{i=0}^{\infty} \epsilon^i \frac{\partial}{\partial x_i^\mu}$$

$$\begin{aligned} A_\mu^a &= A_\mu^a(0) + \epsilon A_\mu^a(1) + \epsilon^2 A_\mu^a(2) + \ldots \\ H &= H(0) + \epsilon H(1) + \epsilon^2 H(2) + \ldots \end{aligned}$$

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- The SU(2)-Higgs field dynamics is described by the Lagrangian:

$$\mathcal{L} = -\frac{1}{4}\mathrm{F}^{\mathrm{a}}_{\mu\nu}\mathrm{F}^{\mathrm{a},\mu\nu} \quad + \quad (\mathrm{D}_\mu\phi)^\dagger(\mathrm{D}^\mu\phi) - \mathrm{V}(\Phi^\dagger\Phi),$$

where:

$$\mathrm{D}_\mu = \partial_\mu + \mathrm{igA}^{\mathrm{a}}_\mu\frac{\sigma^{\mathrm{a}}}{2}, \qquad \mathrm{V}(\Phi^\dagger\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$$\lambda > 0, \qquad \mu^2 < 0 \qquad .$$

- In the standard gauge selection we expand Φ

$$\Phi = \left(0, \frac{1}{\sqrt{2}}(\mathrm{v} + \mathrm{H})\right)^{\mathrm{T}} \quad , \mathrm{v}^2 = -\frac{\mu^2}{\lambda}$$

Equations of motion

$$
\begin{aligned}
(\Box &+ \frac{g^2v^2}{4})A^a_\mu - \partial_\mu(\partial_\nu A^{a,\nu}) + \frac{vg^2}{2}HA^a_\mu + \frac{g^2}{4}H^2A^a_\mu + \\
& g\epsilon_{abc}[(\partial_\mu A^{c,\nu})A^b_\nu - (\partial_\nu A^{b,\nu})A^c_\mu - 2A^b_\nu\partial^\nu A^c_\mu] - \\
& g^2[A^a_\mu A^b_\nu A^{b,\nu} - A^b_\mu A^a_\nu A^{b,\nu}] = 0,
\end{aligned}
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\begin{aligned}
(\Box &+ 2\lambda v^2)H + 3\lambda vH^2 + \lambda H^3 \\
&- \frac{g^2}{4}A^a_\mu A^{a,\mu}(v+H) = 0.
\end{aligned}
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Within the perturbative scheme,equations of motion imply – for reasons of self-consistency and analyticity – that the fields A^a_μ and H should be expanded around the stable minimum , $A^a_\mu(0) = H(0) = 0$.

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- The dynamics of the gauge fields, without Higgs is generally chaotic. (Savvidy et al. 81,Wellner 92,94)
  However,ther exist configurations which admit regular solutions:
  - Color Isotropic Hedgehog ansatz

$$\mathrm{A}_0^a = 0, \qquad \mathrm{A}_i^a = \delta_i^a \mathrm{A}$$

(Savvidy et al. 79; A.Smilga 01; M.Frasca 06,08,09)

- The gauge fields can be expressed as follows:

$$\mathrm{A}_1^1 = \mathrm{A}_2^2 = \mathrm{A}_3^3 = \mathrm{O}(\epsilon) \;\; ; \;\; \mathrm{A}_0^1, \mathrm{A}_0^2, \mathrm{A}_0^3 = \mathrm{O}(\epsilon^2)$$
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- For the Higgs field:
  - 1^{st} case:

    $$\mathrm{H} = 0(\epsilon) \qquad , \qquad \mathrm{H}(1) \neq 0$$

    This case leads to a system of coupled nonlinear partial differential equations of the Nonlinear Schrödinger (NLS)form for which the nonlinear plane wave solutions are unstable for all the values of the parameters.
  - 2^{nd} case:

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    $$(\Box + \mathrm{m}_{\mathrm{A}}^2)\mathrm{A} \quad + \quad \frac{\mathrm{vg}^2}{2}\mathrm{HA} + 2\mathrm{g}^2\mathrm{A}^3 = 0 + \mathrm{O}(\epsilon^4),$$
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    $$\mathrm{m}_{\mathrm{A}}^2 = \frac{\mathrm{g}^2\mathrm{v}^2}{4} \quad , \quad \mathrm{m}_{\mathrm{H}}^2 = 2\lambda\mathrm{v}^2.$$

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The NLS equation for the SU(2) gauge boson

$$\mathrm{O}(\epsilon) : (\Box_0 + \mathrm{m}_\mathrm{A}^2)\mathrm{A}(1) = 0 \qquad \Rightarrow \qquad \mathrm{A}(1) = \mathrm{f}e^{-i\mathrm{m}_\mathrm{A}t} + \mathrm{c.c}$$

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$$\mathrm{f} = \mathrm{f}(\vec{\mathrm{x}}_1, \mathrm{t}_2, ...)$$

In the same order, for the Higgs field:

$$(\Box_0 + \mathrm{m}_\mathrm{H}^2)\mathrm{H}(2) = -\frac{3}{4}\mathrm{v}\mathrm{g}^2\mathrm{A}^2(1) \Rightarrow$$

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where $\mathrm{b} = 2(\mathrm{m}_\mathrm{H}^2 - 4\mathrm{m}_\mathrm{A}^2)/\mathrm{m}_\mathrm{H}^2$ and $\mathrm{B} = -6\mathrm{m}_\mathrm{A}^2/\mathrm{b}\mathrm{v}\mathrm{m}_\mathrm{H}^2$.

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- The function f is determined from the equation of the A(1) field at the order $O(\epsilon^3)$, which leads to the following NLS equation:

$$i\frac{\partial f}{\partial t_2} + \frac{1}{2m_A}\nabla_1^2 f + s|f|^2 f = 0,$$

with $s = \frac{-2g^2(3+\alpha)}{2m_A}$, $\alpha(q) = -\frac{3}{4}\left(\frac{2}{q^2} + \frac{1}{q^2-4}\right)$ and $q = m_H/m_A$.

Stability constraints and the Higgs mass

- The NLS equation possesses exact analytical plane wave solutions of the form:

$$f(\vec{x}_1, t_2) = f_0 \exp[-i(\omega t_2 - \vec{k} \cdot \vec{x}_1)],$$

with the following dispersion relation

$$\omega(k) = \frac{\vec{k}^2}{2m_A} - s|f_0|^2.$$

- The dispersion relation involves an amplitude dependent mass correction. This correction originates from the self-interaction as well as the interaction of the gauge and the Higgs fields, and has in principle similar effect as the quantum self-energy correction.

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Stability of the nonlinear plane waves

- We assume small amplitude perturbations of the form:

$$\delta \mathrm{f} = (\mathrm{u} + \mathrm{iv}) \exp[-\mathrm{i}((\Omega + \omega)\mathrm{t}_2 - \vec{\mathrm{Q}}\vec{\mathrm{x}}_1)]$$

- Then it is straightforward to find from the NLS equation that: Ω and $\vec{\mathrm{Q}}$ satisfy the dispersion relation:

$$\Omega^2 = \frac{|\vec{\mathrm{Q}}|^2}{2\mathrm{m_A}} \left(-2\mathrm{s}|\mathrm{f}_0|^2 + \frac{|\vec{\mathrm{Q}}|^2}{2\mathrm{m_A}} \right).$$

- Hence, the non-linear plane wave solution is stable, i.e., Ω is real for all $|\vec{\mathrm{Q}}|$, only if $\mathrm{s} < 0$,i.e., if the parameter α obeys:

$$3 + \alpha(\mathrm{q}) > 0.$$

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- In the framework of multiscale analysis we obtain restrictions for the Higgs mass originating from stability conditions.

$$56 \text{ GeV} < m_H < 160 \text{ GeV} \quad \text{or} \quad m_H > 165 \text{ GeV}.$$

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  - The nonlinear plane waves are unstable for all parameter values.
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Phys.Rev.D85:027702,2012