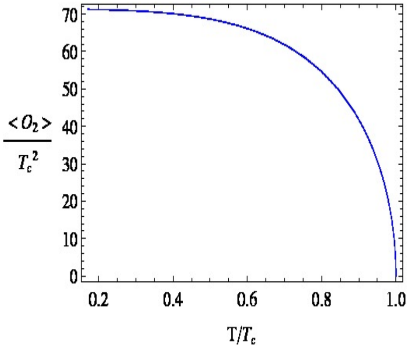


Holographic d-wave superconductors

Marika Taylor

Universiteit van Amsterdam

Introduction

In the last decade a *new conceptual framework* for quantum gravity has emerged:

Holography Any quantum theory of gravity should have a description in terms of a quantum field theory (QFT), *without gravity*, in one dimension less.

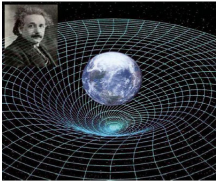
Implications of holography

1. **Fundamental paradigm**: Quantum gravity emerges from conceptually well-understood field theories akin to QCD, with no gravity.

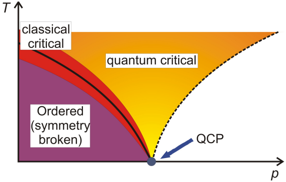
2. **Tool**: Strongly correlated theories can be solved using weakly interacting gravity in one higher dimension!

Tool: modeling quantum critical points

- Much recent interest in engineering geometries to model quantum critical points.



Find geometries with the right properties to model....



(non)-Fermi liquids, (non) relativistic scale invariant systems, cold atoms, **high** T_c **superconductivity....**

References

- “Holographic d-wave superconductors" with K-y. Kim and K. Skenderis, to appear.

Outline

- **Holographic superconductors**
- A new top down approach
- Example: holographic d-wave superconductors

Holographic superconductors

Strong correlations are thought relevant for unconventional superconductors.

But how could we describe superconductors holographically using AdS/CFT?

Holographic superconductors: ingredients

The gravity background must have the following features:

1. **Temperature:** black hole in bulk.
2. **Conserved** $U(1)$ **current**: gauge field A_μ in bulk.
3. **Finite density/doping:** chemical potential μ.
4. **Order parameter:** e.g. charged scalar field ϕ in bulk.

Basic bottom up model

Consider bulk Lagrangian (Gubser; HHH):

$$S = \int d^{d+1}x\sqrt{-g}\left(R - \frac{1}{4}F^2 - |D\phi|^2 - V(\phi)\right).$$

- Above critical temperature T_c: Reissner-Nordstrom black hole.
- Below T_c: black hole with scalar hair is thermodynamically preferred.
- Scalar hair corresponds in dual theory to symmetry breaking operator acquiring expectation value.

Condensed phase in probe approximation

Consider scalar/gauge field as perturbations around AdS-Schwarzschild background:

$$ds^2 = \frac{1}{r^2}\left(-\left(1-\frac{r^d}{r_h^d}\right)dt^2 + \frac{dr^2}{\left(1-\frac{r^d}{r_h^d}\right)} + dx \cdot dx\right).$$

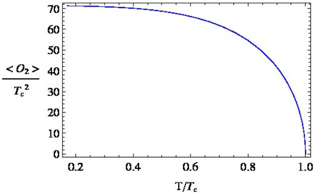
- Above T_c, $\phi = 0$ and $A = \mu\left(1-\frac{r^{d-2}}{r_h^{d-2}}\right)dt$.
- Below T_c, preferred phase has non-trivial profile

$$\phi(r) \qquad A_t(r)$$

with scalar field concentrated near horizon $r = r_h$.

Holographic superconductors

- The operator $\mathcal{O}_2$ dual to the bulk scalar ϕ acquires an expectation value below T_c.
- This vev acts as an order parameter for the $U(1)$ symmetry breaking in d-dimensional field theory.
- There is a second order phase transition at T_c.



Holographic superconductors

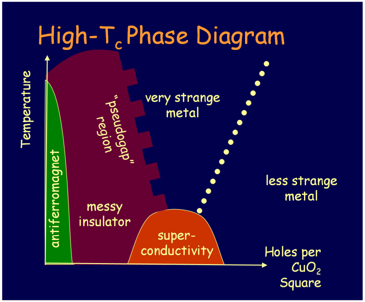
- The superconducting phase is modeled by black hole hair.
- Usually "black holes have no hair" but this is not true for AdS black holes.
- The hair characterizes long range order in the superconductor.

Limitations to bottom up modeling

Simple Lagrangian captures key features but:

1. Lagrangian parameters e.g. scalar potential are arbitrary.
2. Insufficient for analyzing full phase structure and stability.
3. Unconventional superconductors are typically anisotropic, exhibiting p-wave and d-wave superconductivity.

Strange metals and superconductors



(TCM, Cambridge)

- Different bottom up geometries can model holographically parts of the phase diagram.
- E.g. Lifshitz geometries can model strange metal phases (Hartnoll et al, 2010).
- Challenge is to capture all of the diagram in one model.

Outline

- Holographic superconductors
- **A new top down approach**
- Example: holographic d-wave superconductors

Top down approach: consistent truncation

The standard top down approach relies on consistent truncation:

- Reduce and consistently truncate 10/11d supergravity on Sasaki-Einstein manifold.
- Truncated lower-dimensional theory reproduces Lagrangians given earlier.

However:

- Other Kaluza-Klein modes are set to zero, obscuring stability analysis and phase structure.

Consistent truncation is not always possible!

- To model a d-wave superconductor it is natural to use a massive charged spin two field $\phi_{\mu\nu}$ (Benini et al, 2010):

$$S = \int d^{d+1}x\sqrt{-g}[-|D_\rho \phi_{\mu\nu}|^2 + 2|D_\mu \phi^{\mu\nu}|^2 + |D_\mu \phi|^2$$

$$-[D_\mu \phi^{*\mu\nu} D_\nu \phi + \mathrm{c.c.}] - m^2(|\phi_{\mu\nu}|^2 - |\phi|^2)$$

$$+2R_{\mu\nu\rho\lambda}\phi^{*\mu\rho}\phi^{\nu\lambda} - R_{\mu\nu}\phi^{*\mu\lambda}\phi^{\nu}_{\lambda} - \frac{1}{d+1}R|\phi|^2$$

$$-2igqF_{\mu\nu}\phi^{*\mu\lambda}\phi^{\nu}_{\lambda} - \frac{1}{4}F^2]$$

  with m mass, q $U(1)$ charge and g gyromagnetic ratio.
- Such an action, with a finite number of terms, cannot arise from consistent truncation: no-go theorems for massive spin two.

New framework: Kaluza-Klein holography

Work directly with 10d/11d solutions, extracting field theory data with Kaluza-Klein holography techniques. (Skenderis and M.T., 2006)

Kaluza-Klein holography: main idea

1. Consider any 10/11d asymptotically $AdS \times SE$ (Sasaki-Einstein) solution.
2. Near the conformal boundary express the solution as $AdS \times SE$ plus corrections, e.g.

$$ds^2 = ds^2_{AdS}(x^\mu) + ds^2_{SE}(y^a) + h_{mn}(x^\mu, y^a)dx^m dx^n$$

   for the metric perturbations h_{mn}.
3. Express the perturbations in terms of SE harmonics, e.g.

$$h_{\mu\nu} = \sum h^I_{\mu\nu}(x^\mu) Y^I(y^a),$$

   using scalar harmonics $Y^I(y^a)$.

Kaluza-Klein holography: main idea

- Each Kaluza-Klein mode has a mass in AdS, related to the degree of the scalar harmonic.
- Recall the gauge group in AdS corresponds to the isometry group of the SE.
- The charges of the Kaluza-Klein modes correspond to the charges of harmonics under the Killing vectors of the SE.
- Note that $h^{I}_{\mu\nu}$ correspond to massive spin two modes in AdS.

Kaluza-Klein holography: dual field theory

- Expressing the AdS metric as

$$ds^2 = \frac{dr^2}{r^2} + \frac{dx^i dx^i}{r^2}$$

  with the conformal boundary as $r \to 0$, for each Kaluza-Klein mode we can look at the asymptotic expansion near $r \to 0$ and read off the dual field theory data.

- For example, for a scalar mode φ dual to an operator of dimension Δ then

$$\varphi \sim r^{d-\Delta}(\varphi_s + \mathcal{O}(r))$$
$$+ r^{\Delta}(\tilde{\varphi} + \mathcal{O}(r)),$$

  with φ_s the source for the operator and its vev related to $\tilde{\varphi}$.

Kaluza-Klein holography

Consider for definiteness the case of $AdS_5 \times S^5$.

- The effective 5d action involves an infinite number of KK modes, interacting with each other.
- **Q:** So how can one work with this action, without consistently truncating?
- **A:** To extract holographic information about an operator of given dimension, one needs only retain a finite number of fields, dual to operators of this dimension or less.

Kaluza-Klein holography

Any Kaluza-Klein mode, including massive spin two fields, can be switched on, with the relevant effective action systematically determined, iteratively!

Outline

- Holographic superconductors
- A new top down approach
- **Example: holographic d-wave superconductors**

Holographic d-wave superconductors

Consider probe limit around Schwarzschild $AdS_5 \times S^5$:

1. Switch on metric perturbation $h_{\mu\nu} = \phi_{\mu\nu}(x^\mu) Y^I(\theta)$, corresponding to a massive spin two.
2. Switch on metric/five form perturbations $A_\mu Y^a(\theta)$, with Y^a a Killing vector of S^5, corresponding to 5d gauge field.
3. The spin two field $\phi_{\mu\nu}$ is charged under this gauge field.

Similarly analysis for Schwarzschild $AdS_4 \times S^7$, for dual three-dimensional theory.

Reduced action

- The effective action for these fields is

$$S = \int d^5x \sqrt{-g}[-|D_\rho \phi_{\mu\nu}|^2 + 2|D_\mu \phi^{\mu\nu}|^2 + |D_\mu \phi|^2$$
$$-[D_\mu \phi^{*\mu\nu} D_\nu \phi + \mathrm{c.c.}] - m^2(|\phi_{\mu\nu}|^2 - |\phi|^2)$$
$$-2igqF_{\mu\nu}\phi^{*\mu\lambda}\phi^{\nu}_{\lambda} - \frac{1}{4}F^2 + \cdots]$$

with m mass, q $U(1)$ charge and g gyromagnetic ratio all **determined** by the 10d equations.

Condensate phase

Recall the AdS-Schwarzschild background:

$$ds^2 = \frac{1}{r^2}\left(-\left(1 - \frac{r^d}{r_h^d}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r^d}{r_h^d}\right)} + dx \cdot dx \right).$$

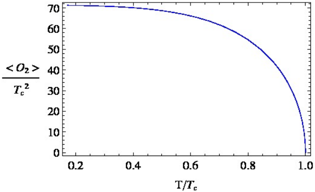
- Above T_c, $\phi_{\mu\nu} = 0$ and $A = \mu \left(1 - \frac{r^{d-2}}{r_h^{d-2}}\right) dt$.
- Below T_c, there is a condensed phase

$$\phi_{xy}(r) \qquad A_t(r)$$

with the spin two field breaking the spatial isotropy.

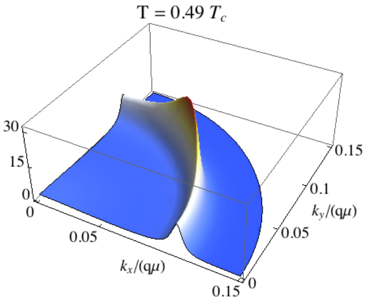
Holographic d-wave superconductors

- The operator $\mathcal{O}_{xy}$ acquires an expectation value below T_c.
- This vev acts as an order parameter for the $U(1)$ symmetry breaking and the breaking of isotropy in dual 4 dimensional field theory.



ARPES spectra

- One can also compute fermionic correlation functions in the condensed background.
- These relate to experimentally measured ARPES spectra.
- Fermi arcs in top down models?



Summary

- Anisotropic superconducting phases are holographically modeled by anisotropic black hole hair.
- Massive spin two KK excitations are condensed in d-wave superfluids.
- Switching on other KK excitations, such as massive vectors, can produce a rich landscape of p-wave and other phases.

Outlook and challenges

1. How well do our top down d-wave models match with unconventional superconductors?
2. Can one obtain a realistic superconductor phase diagram, using other KK modes?
3. Holographic systems are always large N: beyond classical gravity?
4. Always superfluid: global $U(1)$ needs to be weakly gauged.