The holographic fluid dual to flat spacetime

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Holography

- Any gravitational theory is expected to be holographic, *i.e.* it should have a description in terms of a non-gravitational theory in one dimension less.
- If gravity is indeed holographic, one should be able to recover generic features of quantum theories through gravitational computations.
- One of the most basic such features is the UV behavior of the quantum theory: the UV divergences of a local QFT are local.
- Via the UV/IR connection, any gravitational theory dual to a local QFT must have local IR divergences.

Holography and asymptotics

- Indeed, in the cases we understand holography, *i.e.* for asymptotically AdS spacetimes and spacetimes conformal to that, one can prove that the divergences are local in boundary data. [Henningson, KS (1998)], [Kanitscheider, KS, Taylor (2008)]
- Conversely, if the IR divergences of a gravitational theory are non-local, the dual quantum theory cannot be a local QFT.
- Asymptotically flat spacetimes fall into this category. The structure of the asymptotic solutions shows that the divergences of the on-shell action are non-local in boundary data. [de Haro, Solodukhin, KS (2001)].
- Holography for such spacetimes is more difficult to understand ... as the dual theory should be non-local.

Holography and long wavelength behavior

- Another generic feature of QFTs is the existence of a *hydrodynamic description* capturing the long-wavelength behavior near to thermal equilibrium.
- One then expects to find the same feature on the gravitational side, *i.e.*, there should exist a bulk solution corresponding to the thermal state, and nearby solutions corresponding to the hydrodynamic regime.
- Global solutions corresponding to non-equilibrium configurations should be well-approximated by the solutions describing the hydrodynamic regime at sufficiently long distances and late times.

Hydrodynamics and AdS/CFT

This picture is indeed beautifully realized in AdS/CFT:

Thermal state	$\Leftrightarrow$	AdS black hole
Relativistic hydrodynamics of a conformal fluid	$\Leftrightarrow$	Bulk solution in a relativistic gradient expansion

- Solutions describing non-equilibrium configurations are well approximated by hydrodynamics at late times.

[Witten (1998)] ... [Policastro, Son, Starinets (2001)] ... [Janik, Peschanski (2005)] ... [Bhattacharyya, Hubeny, Minwalla, Rangamani (2007)] ... [Chestler, Yaffe (2010)] ...

Hydrodynamics and vacuum Einstein gravity

We will see that a similar picture can be developed for vacuum Einstein gravity:

Thermal state	$\Leftrightarrow$	Rindler space
Relativistic hydrodynamics of a "peculiar" fluid	$\Leftrightarrow$	Bulk solution in a relativistic gradient expansion

One may then use the properties of these solutions in order to obtain clues about the nature of the dual theory.

References

- The talk is based on

  Geoffrey Compère, Paul McFadden, KS, Marika Taylor,
  The relativistic fluid dual to vacuum Einstein gravity,
  [arXiv:1201.2678].
  The holographic fluid dual to vacuum Einstein gravity,
  [arXiv:1104.3894].

- Related work

  C. Eling, A. Meyer, Y. Oz, arXiv:1201.2705.

  I. Bredberg, C. Keeler, V. Lysov, A. Strominger,
  [arXiv:1101.2451]; V. Lysov, A. Strominger [arXiv:1104.5502].

References

- Earlier work

T. Damour, PhD thesis, 1979; K. Thorne, R. Prince, D. Macdonald, “Black Holes: the membrane paradigm" (1986).

I. Fouxon, Y. Oz, [arXiv:0809.4512]; C. Eling, I. Fouxon, Y. Oz, [arXiv:0905.3638].

S. Bhattacharyya, S. Minwalla, S. Wadia, [arXiv: 0810.1545].

I. Bredberg, C. Keeler, V. Lysov, A. Strominger, [arXiv:1006.1902].

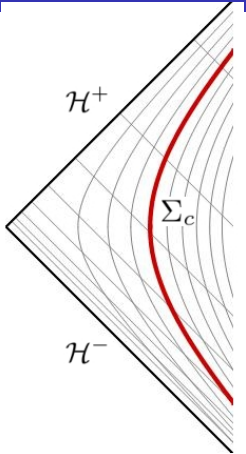
Rindler spacetime

- Flat spacetime in ingoing Rindler coordinates is give by:

$$ds^2 = -rd\tau^2 + 2d\tau dr + dx_i dx^i$$

  i.e. Minkowski space parametrised by timelike hyperbolae $X^2 - T^2 = 4r$ and ingoing null geodesics $X + T = e^{\tau/2}$.

- We will consider the portion of spacetime between $r = r_c$ and the future horizon, $\mathcal{H}^+$, the null hypersurface $X = T$.



Equilibrium configurations

We now want to obtain a family of equilibrium configurations parametrized by arbitrary constants that would become the hydrodynamic variables in the hydrodynamic regime.

We require three properties:

1. There exists a co-dimension one hypersurface Σ_c on which the fluid lives, with flat induced metric:

$$\gamma_{ab} dx^a dx^b = -r_c d\tau^2 + dx_i dx^i$$

$\sqrt{r_c}$ is speed of light (arbitrary)

2. The Brown-York stress tensor on Σ_c takes the perfect fluid form

$$T_{ab} = \rho u_a u_b + p h_{ab},$$

where $h_{ab} = \gamma_{ab} + u_a u_b$ is spatial metric in local rest frame of fluid.

3. Stationary w.r.t. ∂_τ and homogeneous in x^i directions.

Equilibrium configurations

- One configuration satisfying properties ❶, ❷, ❸ is Rindler spacetime.
- We generate metrics with arbitrary constant p and u^a by acting on Rindler spacetime with diffeomorphisms.
- There are the only *two infinitesimal diffeomorphisms* that preserve the properties ❶, ❷, ❸.

Equilibrium configurations

Applying these two transformations, the resulting metric is

$$ds^2 = -p^2(r - r_c)u_a u_b dx^a dx^b - 2pu_a dx^a dr + \gamma_{ab} dx^a dx^b.$$

- The induced metric on Σ_c is still γ_{ab}.
- The Brown-York stress tensor is that of a perfect fluid with

$$\rho = 0, \qquad p = 1/\sqrt{r_c - r_h}, \qquad u^a = 1/\sqrt{r_c - v^2}(1, v_i).$$

- The Unruh temperature is given by

$$T = 1/(4\pi\sqrt{r_c - r_h})$$

and all thermodynamic identities are satisfied, with the entropy density given by $s = 1/4G$.

From equilibrium to hydrodynamics

We now wish to consider near-equilibrium configurations.

- We consider the pressure field p and velocities u_a as slowing varying functions of the coordinates.
- We consider a standard relativistic derivative expansion in the boundary directions,

$$\partial_r \sim 1, \qquad \partial_a \sim \epsilon,$$

and solve Einstein equations iteratively.

Solution to all orders

One can construct the solution to arbitrarily high order in ϵ.

- Suppose we have a solution at order ϵ^{n-1}. Let's now add a new term to the metric $g_{\mu\nu}^{(n)}$ at order ϵ^n. The Ricci tensor is

$$R_{\mu\nu}^{(n)} = \delta R_{\mu\nu}^{(n)} + \hat{R}_{\mu\nu}^{(n)}.$$

Here, $\delta R_{\mu\nu}^{(n)}$ is the contribution at order ϵ^n due to the new term $g_{\mu\nu}^{(n)}$, while $\hat{R}_{\mu\nu}^{(n)}$ is the nonlinear contribution at order ϵ^n from the metric at lower orders.

- ⇒ We now set $R_{\mu\nu}^{(n)} = \delta R_{\mu\nu}^{(n)} + \hat{R}_{\mu\nu}^{(n)} = 0$ and solve for $g_{\mu\nu}^{(n)}$ in terms of the metric at lower orders.

Integration scheme

- We choose the gauge

$$g_{rr} = 0, \qquad g_{ra} = -pu_a$$

The ab metric components may be decomposed as

$$g^{(n)}_{ab} = \alpha^{(n)} u_a u_b + 2\beta^{(n)}_{(a} u_{b)} + \gamma^{(n)}_{ab} .$$

where $\alpha^{(n)}, \beta^{(n)}, \gamma^{(n)}$ are determined by solving the equations.

- Furthermore, we impose boundary conditions such that:
  - the metric on Σ_c is preserved
  - the solution is regular on the future horizon $\mathcal{H}^+$.

Solution

- Our final integration scheme is thus

$$\alpha^{(n)} = (1-r)F^{(n)}(x) + 2p^2 \int_r^1 dr' \int_{r'}^1 dr'' (h^{cd}\hat{R}^{(n)}_{cd} - \frac{1}{2}\hat{R}^{(n)}),$$

$$\beta^{(n)}_a = (1-r)F^{(n)}_a(x) + 2p \int_r^1 dr' \int_{r'}^1 dr'' h^b_a \hat{R}^{(n)}_{br},$$

$$\gamma^{(n)}_{ab} = -2 \int_r^1 dr' \frac{1}{r'-r_{\mathcal{H}}} \int_{r_{\mathcal{H}}}^{r'} dr'' h^c_a h^d_b \hat{R}^{(n)}_{cd}.$$

where the arbitrary functions $F^{(n)}$ and $F^{(n)}_a$ encode the gauge freedom for the dual fluid at order ϵ^n.

Fluid gauge conditions

The remaining freedom may be fixed by choosing appropriate gauge conditions for the dual fluid.

- $F_a^{(n)}$ may be fixed by imposing gauge condition:

$$0 = u^a T_{ab} h_c^b \quad \Rightarrow \quad F_a^{(n)} = p h_a^b \hat{T}^{\nu}_{bc} u^c .$$

  i.e. the momentum density $T_{\tau i}$ vanishes in the local rest frame. This is effectively a definition of the fluid velocity u^a.

- $F^{(n)}$ is fixed by imposing that there are no corrections to the pressure.
- With all gauge freedom now fixed, we have a unique solution for the bulk metric.

The dual stress energy tensor

One can now extract from the solution the dual stress energy tensor.

- A relativistic fluid stress tensor can be written in the form:

$$T_{ab} = \rho u_a u_b + p h_{ab} + \Pi^{\perp}_{ab}, \qquad u^a \Pi^{\perp}_{ab} = 0,$$

  where $\Pi^{\perp}_{ab}$ represents dissipative corrections and may be expanded in fluid gradients.

- The 'generalized' equation of state $dT_{ab}T^{ab} = T^2$ determines the energy density ρ in terms of p and $\Pi^{\perp}_{ab}$, so we only need determine $\Pi^{\perp}_{ab}$.

Second-order relativistic hydrodynamics

To second order in gradients, $\Pi^{\perp}_{ab}$ is given by the following expansion

$$\Pi^{\perp}_{ab} = -2\eta\mathcal{K}_{ab} + \frac{1}{p}\Big(c_1\mathcal{K}^c_a\mathcal{K}_{cb} + c_2\mathcal{K}^c_{(a}\Omega_{|c|b)} + c_3\Omega_a{}^c\Omega_{cb} + c_4 h^c_a h^d_b \partial_c\partial_d \ln p + c_5\mathcal{K}_{ab}\, D\ln p + c_6 D^{\perp}_a \ln p\, D^{\perp}_b \ln p\Big) + O(\partial^3),$$

where $D^{\perp}_a = h^b_a\partial_b$ and $D = u^a\partial_a$ and the vorticity $\Omega_{ab} = h^c_a h^d_b \partial_{[c}u_{d]}$.

- There is one first order transport coefficient, the shear viscosity η and six second-order transport coefficients: c_1, c_2, etc.

Characterizing the dual fluid

- Extracting T_{ab} from the geometry we find that it takes the expected form and we read-off the transport coefficients:

$$\eta = 1, \qquad c_1 = -2,\ c_2 = c_3 = c_4 = c_5 = -c_6 = -4.$$

- The shear viscosity saturates the KSS bound,

$$\eta/s = 1/(4\pi)$$

Entropy current

- A general feature of systems away from equilibrium is that they possess an entropy current with non-negative divergence. This encodes the second law of thermodynamics.
- At equilibrium this current reduces to the conserved entropy current,

$$J^a_{eq} = s_{eq} u^a,$$

where s_{eq} the entropy density at equilibrium.
- In the hydrodynamic regime, the entropy current may differ from this expression by terms of higher order in gradients.

Classification of entropy current to second order

- There is a five-parameter family of allowed entropy currents (in flat spacetime):

$$J^a = s_{eq} u^a \left(1+\frac{1}{p^2}\left(a_1 \mathcal{K}_{ab}\mathcal{K}^{ab}+a_2 \Omega_{ab}\Omega^{ab}+a_3(D_\perp \ln p)^2\right)\right)+\frac{s_{eq}}{p^2}\left(b_1 h^{ac}\partial_b \mathcal{K}^b_c\right.$$
$$\left.+b_2 D^a_\perp D\ln p + b_3 \mathcal{K}^a_b D^b_\perp \ln p + b_4 \Omega^{ab} D^\perp_b \ln p + b_5 D^a_\perp \ln p D\ln p\right).$$

where

$$a_3=(5b_1-4a_2-4b_2-b_3)/2,\ b_4=2b_1-2b_2-b_3,\ b_5=4a_2-5b_1+3b_2+b_3.$$

- A two-parameter subset of these entropy currents is in fact trivially conserved, i.e. they are of the form $j^a = \partial_b \chi^{[ab]}$.

Entropy current from gravity

We now want to construct an entropy current holographically, following the AdS construction of [Bhattacharyya etal, 0803.2526]

- In equilibrium, the entropy is represented by the area of the black hole horizon, thus the holographic entropy current should be closely related with the black hole horizon.
- The area increase theorem of general relativity should then imply the non-negativity of the divergence of the entropy current.

The holographic entropy current

Indeed we showed, extending [Booth etal 1010.6301], that the current

$$J^a = \frac{1}{4G_N} \frac{\sqrt{-g_{\mathcal{H}}}}{\sqrt{-g_{\Sigma}}} \xi^a_{\mathcal{H}}$$

has non-negative divergence due to the Raychaudhuri equation.

- $\xi^a_{\mathcal{H}}$ is a (non-affine) horizon generator.
- $\sqrt{-g_{\mathcal{H}}}$ is the area of the spatial sections of the horizon.
- $\sqrt{-g_{\Sigma}}$ is the area of the spatial sections of Σ_c.

Evaluating the entropy current

Evaluating this formula we obtain

$$J^a = 4\pi u^a \left(1 + \frac{1}{p^2}\mathcal{K}_{bc}\mathcal{K}^{bc} + \frac{1}{2p^2}\Omega_{bc}\Omega^{bc}\right)$$
$$+ \frac{4\pi}{p^2}\left(2D_\perp^a \ln p D \ln p - 2D_\perp^a D \ln p - h^{ab}\pi_c\mathcal{K}_b^c + (\mathcal{K}_b^a + \Omega^a{}_b)D_\perp^b \ln p\right) + O(\partial^3).$$

which is of the form we found earlier with

$$a_1 = 1, \qquad a_2 = \frac{1}{2}, \qquad b_1 = -1, \qquad b_2 = -2, \qquad b_3 = 1.$$

- Given the unconventional properties of the fluid dual to vacuum Einstein gravity, it is reassuring that the entropy current indeed has non-negative divergence.

A model for the dual fluid

- We now propose a simple Lagrangian model for the dual fluid. We focus on the non-dissipative part of the stress tensor,

$$T_{ab} = ph_{ab} = p(\gamma_{ab} + u_a u_b),$$

describing a fluid with nonzero pressure but vanishing energy density in the local rest frame.

- To get the dissipative part would need to couple to a heat bath.

A model for the dual fluid

$$S = \int d^{d+1}x\sqrt{-\gamma}\sqrt{-(\partial\phi)^2}.$$

- The field equations describe *potential flow*

$$\nabla^a u_a = 0, \qquad u_a = \frac{\partial_a\phi}{\sqrt{X}}, \qquad X = -(\partial\phi)^2.$$

- The stress tensor is

$$T_{ab} = \sqrt{X}\gamma_{ab} + \frac{1}{\sqrt{X}}\partial_a\phi\partial_b\phi = \sqrt{X}h_{ab}, \qquad \text{i.e.} \quad p = \sqrt{X}.$$

reproducing the holographic result.

Conclusions

- ➢ We've established a direct relation between $(d+2)$-dimensional Ricci-flat metrics and $(d+1)$-dimensional fluids satisfying the incompressible relativistic Navier-Stokes equations.

- ➢ The dual fluid has vanishing equilibrium energy density but nonzero pressure. We computed the viscosity and the six second-order transport coefficients and we obtained a holographic entropy current.

- ➢ A simple sqrt Lagrangian captures the non-dissipative properties of the fluid.

Outlook

- Does the correspondence extend beyond the hydrodynamic regime on the field theory side, and/or the classical gravitational description on the bulk side? Is there a string embedding? Can we get the sqrt action from branes?
- How far can flat space holography be developed? Is there a holographic dictionary relating bulk computations to quantities in the dual field theory on Σ_c?
- By the equivalence principle, our construction should hold locally in any small neighbourhood. Can one patch together such a 'local' holographic description of neighbourhoods to obtain a global holographic description of general spacetimes?