Model Building with F-Theory GUTs

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Outline

- Introduction and motivation
- Local F-theory models
- Model which stabalises the proton and forbids R-parity violating operators
- Issues to be addressed

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- A class of models which have these properties are F-theory GUTs

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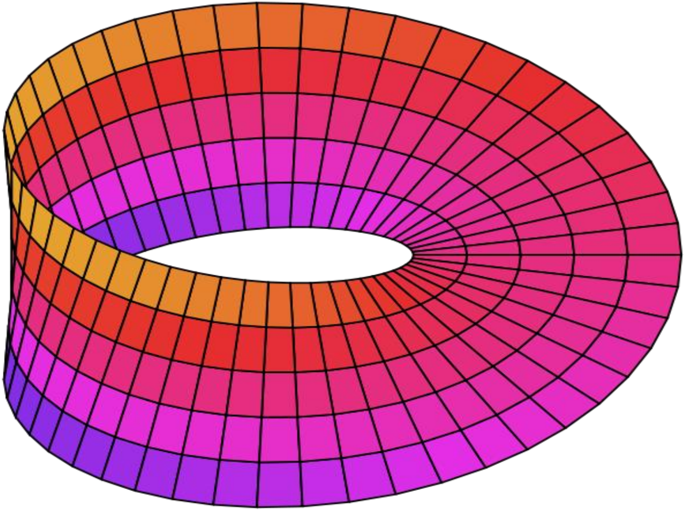
- The 12 dimensional theory can be broken up into the 4 large spacetime dimensions, 6 internal dimensions, and 2 additional dimensions which encode the variation of the real and imaginary parts of the axio-dilaton
- The formal language of global F-theory models is that of an elliptically fibred Calibi-Yau fourfold, X, with a threefold base, B_3

Moebius Strip- Example of a Fibre Bundle



- The form of the elliptic fibration is

$$y^2 + \alpha_1 xy + \alpha_3 y = x^3 + \alpha_2 x^2 + \alpha_4 x + \alpha_6$$

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- To get a local model, we assign scaling dimensions to the coordinates, and discard irrelevant terms, eg.

$$E_6 : y^2 = x^3 + b_3 y z^2 + b_2 x z^3 + b_0 z^5$$

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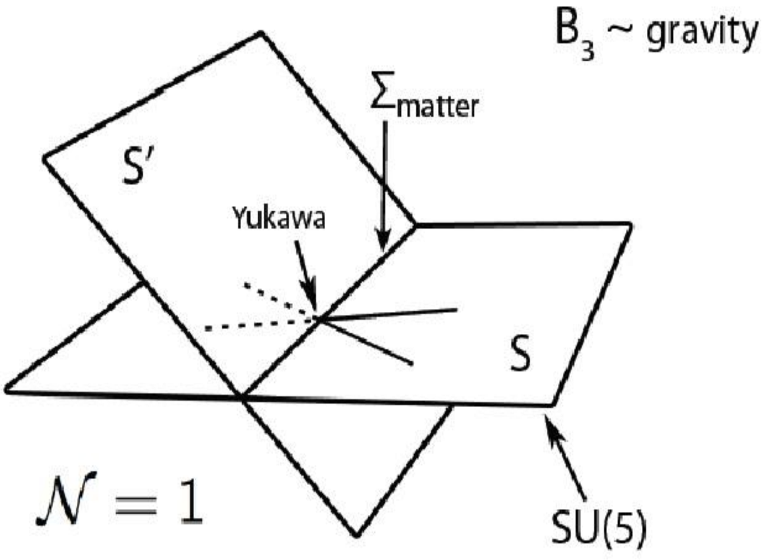
- Different view: larger gauge group, broken by a position dependent VEV for an adjoint Higgs field


$B_3 \sim$ gravity
Σ_{matter}
S'
Yukawa
S
$\mathcal{N} = 1$
SU(5)

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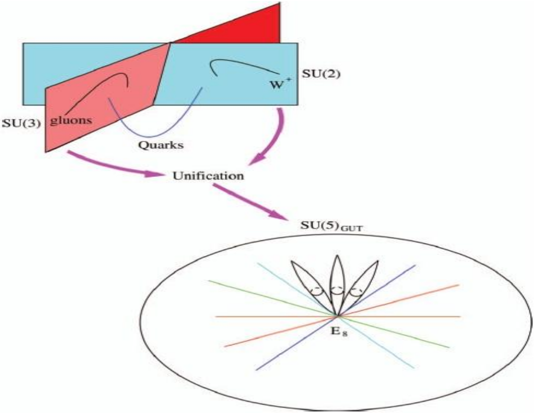

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- The equations for the curves in terms of the weights t_i ($i = 1, .., 5$, $\sum t_i = 0$), of the 5 representation of SU(5) are

$$\Sigma_{10} : t_i = 0$$
$$\Sigma_5 : -t_i - t_j = 0, i \neq j$$
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- Non linear $\to$ Monodromies

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Field	$SU(5) \times SU(5)_\perp$	$SU(5)_\perp$ component
$Q_3,\ U_3^c,\ l_3^c$	(10,5)	$t_{1,2}$
$Q_2,\ U_2^c,\ l_2^c$	(10,5)	t_3
$Q_1,\ U_1^c,\ l_1^c$	(10,5)	t_4
$D_3^c,\ L_3$	$(\overline{5}, 10)$	$t_3 + t_5$
$D_2^c,\ L_2$	$(\overline{5}, 10)$	$t_1 + t_3$
$D_1^c,\ L_1$	$(\overline{5}, 10)$	$t_1 + t_4$
H_u	$(5, \overline{10})$	$-t_1 - t_2$
H_d	$(\overline{5}, 10)$	$t_1 + t_4$
θ_{ij}	(1,24)	$t_i - t_j$

Table: *Matter Fields and their assignments (King, Leontaris, Ross)*

Family Symmetries

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- The up quark Yukawa operators $Q_i U^c_j H_u$ require the following flavons to balance the charges
- $Q_i U^c_j H_u \begin{pmatrix} \theta_{14}^2 & \theta_{13}\theta_{14} & \theta_{14} \\ \theta_{13}\theta_{14} & \theta_{13}^2 & \theta_{13} \\ \theta_{14} & \theta_{13} & 1 \end{pmatrix} \rightarrow Y^u \sim \begin{pmatrix} \epsilon^6 & 3\epsilon^5 & \epsilon^3 \\ 3\epsilon^5 & 9\epsilon^4 & 3\epsilon^2 \\ \epsilon^3 & 3\epsilon^2 & 1 \end{pmatrix}$

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- The down quark Yukawa operators $Q_i D^c_j H_d$ require the following flavons to balance the charges
- $Q_i D^c_j H_d \begin{pmatrix} \theta_{54}\theta_{34} & \theta_{54} & \theta_{14} \\ \theta_{54} & \theta_{53} & \theta_{13} \\ \theta_{31}\theta_{54} + \theta_{34}\theta_{51} & \theta_{23} & \theta_{23} \end{pmatrix} \rightarrow Y^d \sim \begin{pmatrix} 0 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & 3\epsilon^2 \\ 0 & 0 & 1 \end{pmatrix}$
- The additional choices $\theta_{53} = \epsilon^2, \quad \theta_{54} = \epsilon^3, \quad \theta_{31} = 0$ have been made

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- Fluxes inside GUT group

Flux breaking

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Flux can be used to break down to the SM, leading to splitting equations

$$10 = \begin{cases} \text{Rep.} & \# \\ n^1_{3,2} - n^1_{\bar{3},2} & : \ M_{10} \\ n^1_{\bar{3},1} - n^1_{3,1} & : \ M_{10} - N \\ n^1_{\bar{1},1} - n^1_{1,1} & : \ M_{10} + N \end{cases} \qquad 5 = \begin{cases} \text{Rep.} & \# \\ n^1_{3,1} - n^1_{\bar{3},1} & : \ M_5 \\ n^1_{1,2} - n^1_{1,\bar{2}} & : \ M_5 + N \end{cases}$$

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The Ms and Ns are integers given by the flux dotted with the homology class of the matter curve.

The flux associated with M respects the GUT structure, and so is a flux in the perpendicular U(1)s.

The flux associated with N is a hypercharge flux, and leads to incomplete SU(5) multiplets.

E_6 Origin of $SU(5)$ Matter	$U(1)_i$	homology	$U(1)_Y$-flux	$U(1)$-flux
$27_{t_1} \supset 16_{t_1} \supset 10_M$	$t_{1,2}$	$\eta - 2c_1 - \chi$	$-N$	M_{10_1}
$27_{t_3} \supset 16_{t_3} \supset 10_2$	t_3	$-c_1 + \chi_5$	N_5	M_{10_2}
$78 \supset 16_{t_4} \supset 10_3$	t_4	$-c_1 + \chi_7$	N_7	M_{10_3}
$78 \supset 45_{t_5} \supset 10_4$	t_5	$-c_1 + \chi_9$	N_9	M_{10_4}
$27_{t_3} \supset 10_{t_3} \supset 5_{H_u}$	$-t_1 - t_2$	$-c_1 + \chi$	N	$M_{5_{h_u}}$
$27_{t_1} \supset 10_{t_1} \supset 5_1$	$-t_{1,2} - t_3$	$\eta - 2c_1 - \chi$	$-N$	M_{5_1}
$27_{t_1} \supset 10_{t_1} \supset 5_2$	$-t_{1,2} - t_4$	$\eta - 2c_1 - \chi$	$-N$	M_{5_2}
$27_{t_1} \supset 16_{t_1} \supset 5_3$	$-t_{1,2} - t_5$	$\eta - 2c_1 - \chi$	$-N$	M_{5_3}
$27_{t_3} \supset 10_{t_3} \supset 5_4$	$-t_3 - t_4$	$-c_1 + \chi - \chi_9$	$N - N_9$	M_{5_4}
$27_{t_3} \supset 16_{t_3} \supset 5_5$	$-t_3 - t_5$	$-c_1 + \chi - \chi_7$	$N - N_7$	$M_{5_{h_d}}$
$78 \supset 16_{t_4} \supset 5_6$	$-t_4 - t_5$	$-c_1 + \chi - \chi_5$	$N - N_5$	M_{5_6}

Table: *Field representation content under* $SU(5) \times U(1)_{t_i}$*, their homology class and flux restrictions. For convenience, only the properties of* $10, 5$ *are shown.* $\overline{10}, \overline{5}$ *are characterized by opposite values of* $t_i \to -t_i$ *etc. Note that the fluxes satisfy* $N = N_5 + N_7 + N_9$ *and* $\sum_i M_{10_i} + \sum_j M_{5_j} = 0$ *while* $\chi = \chi_5 + \chi_7 + \chi_9$.

We know about the origins of the singlets also

Singlet	Q_χ	Q_ψ	Representations
θ_{12}	0	0	SO(10) singlet in 78
θ_{13}	0	0	$45 \subset 78$
θ_{14}	0	4	SO(10) singlet in $27_{t_{1,2}}$
θ_{15}	-5	1	$16_{t_{1,2}} \subset 27_{t_{1,2}}$
θ_{34}	0	4	SO(10) singlet in 27_{t_3}
θ_{35}	-5	-1	$16_{t_3} \subset 27_{t_3}$
θ_{45}	-5	-3	$16_{t_4} \subset 78$

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E_6, $SO(10)$, $SU(5)$	Charge	N_Y	$M_{U(1)}$	SM particle content
27_{t_1}, 16_{t_1}, $\overline{5}_3$	t_1+t_5	$\tilde{N}$	$-M_{5_3}$	$-M_{5_3}d^c+(-M_{5_3}+\tilde{N})L$
27_{t_1}, 16_{t_1}, 10_M	t_1	$-\tilde{N}$	$-M_{5_3}$	$-M_{5_3}Q+(-M_{5_3}+\tilde{N})u^c+(-M_{5_3}-\tilde{N})e^c$
27_{t_1}, 16_{t_1}, θ_{15}	t_1-t_5	0	$-M_{5_3}$	$-M_{5_3}\nu^c$
27_{t_1}, 10_{t_1}, 5_1	$-t_1-t_3$	$-\tilde{N}$	$-M_{5_3}$	$-M_{5_3}D+(-M_{5_3}-\tilde{N})H_u$
27_{t_1}, 10_{t_1}, $\overline{5}_2$	t_1+t_4	$\tilde{N}$	$-M_{5_3}$	$-M_{5_3}\overline{D}+(-M_{5_3}+\tilde{N})H_d$
27_{t_1}, 1_{t_1}, θ_{14}	t_1-t_4	0	$-M_{5_3}$	$-M_{5_3}S$
27_{t_3}, 16_{t_3}, $\overline{5}_5$	t_3+t_5	$-\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}d^c+(M_{5_{H_u}}-\tilde{N})L$
27_{t_3}, 16_{t_3}, 10_2	t_3	$\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}Q+(M_{5_{H_u}}-\tilde{N})u^c+(M_{5_{H_u}}+\tilde{N})e^c$
27_{t_3}, 16_{t_3}, θ_{35}	t_3-t_5	0	$M_{5_{H_u}}$	$M_{5_{H_u}}\nu^c$
27_{t_3}, 10_{t_3}, 5_{H_u}	$-2t_1$	$\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}D+(M_{5_{H_u}}+\tilde{N})H_u$
27_{t_3}, 10_{t_3}, $\overline{5}_4$	t_3+t_4	$-\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}\overline{D}+(M_{5_{H_u}}-\tilde{N})H_d$
27_{t_3}, 1_{t_3}, θ_{34}	t_3-t_4	0	$M_{5_{H_u}}$	$M_{5_{H_u}}S$

Spectrum

Spectrum

- Choosing the case $M_{5_3} = -3$, $M_{5_{H_u}} = 0$ and $\tilde{N} = 1$, the spectrum is
- $3[Q, u^c, d^c, L, e^c, \nu^c]_{16}$
- $3[H_u, D, H_d, \overline{D}]_{10}$
- $3[S]_1$
- $L + \overline{L}, e^c + \overline{e^c}, u^c + \overline{u^c}, H_d + \overline{H_d}$

Spectrum

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- $3[Q, u^c, d^c, L, e^c, \nu^c]_{16}$
- $3[H_u, D, H_d, \overline{D}]_{10}$
- $3[S]_1$
- $L + \overline{L}, e^c + \overline{e^c}, u^c + \overline{u^c}, H_d + \overline{H_d}$
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- With this choice, however, H_d comes from 27_{t_1} and so down quark masses are forbidden at tree level!
- $\Rightarrow$ Slightly modified structure:

$$\begin{aligned} M_{10_M} &= -M_{5_3} = 4, \\ M_{5_1} &= -M_{5_2} = 3 \\ M_{10_2} &= -M_{5_5} = -1, \\ M_{5_4} &= M_{H_u} = 0, \\ M_{\theta_{15}} &= 2, \\ \tilde{N} &= 1 \end{aligned}$$

E_6	$SO(10)$	$SU(5)$	N_Y	$M_{U(1)}$	SM particle content	Low energy
$27_{t_1'}$	16	$\overline{5}_3$	1	4	$4d^c + 5L$	$3d^c + 3L$
$27_{t_1'}$	16	10_M	-1	4	$4Q + 5u^c + 3e^c$	$3Q + 3u^c + 3e^c$
$27_{t_1'}$	16	θ_{15}	0	3	$3\nu^c$	-
$27_{t_1'}$	10	5_1	-1	3	$3D + 2H_u$	-
$27_{t_1'}$	10	$\overline{5}_2$	1	3	$3\overline{D} + 4H_d$	H_d
$27_{t_3'}$	16	$\overline{5}_5$	-1	-1	$\overline{d^c} + 2\overline{L}$	-
$27_{t_3'}$	16	10_2	1	-1	$\overline{Q} + 2\overline{u^c}$	-
$27_{t_3'}$	16	θ_{35}	0	0	$-$	-
$27_{t_3'}$	10	5_{H_u}	1	0	H_u	H_u
$27_{t_3'}$	10	$\overline{5}_4$	-1	0	$\overline{H_d}$	-
$27_{t_3'}$	1	θ_{34}	0	1	θ_{34}	-
-	1	θ_{31}	0	4	θ_{31}	-
-	1	θ_{53}	0	1	θ_{53}	-
-	1	θ_{14}	0	3	θ_{14}	-
-	1	θ_{45}	0	2	θ_{45}	-

- The vector pairs with components in the 27_{t_1} and 27_{t_3} multiplets are removed by introducing θ_{31} which is a singlet of E_6 and has couplings:

$$\theta_{31} 27_{t_1'} \overline{27_{t_3'}} = \theta_{31} Q\overline{Q} + \theta_{31}(2u^c)(2\overline{u^c}) + \theta_{31} d^c\overline{d^c} + \theta_{31}(2L)(2\overline{L}) + \theta_{31} H_d\overline{H_d}$$

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- If θ_{31} gets a large VEV these vector states get large masses as required.
- To remove the remaining exotics we introduce θ_{34} which has the couplings :

$$\theta_{34}5_1\overline{5_2} = \theta_{34}[3D+2H_u][3\overline{D}+3H_d] = \theta_{34}[3(D\overline{D})] + \theta_{34}[2(H_uH_d)]$$

If it too acquires a large VEV it generates large mass to the three copies of $D+\overline{D}$ (solving the doublet-triplet splitting problem) and two families of Higgs H_u, H_d, leaving just the MSSM spectrum

Singlet VEVs

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- To determine the large singlet VEVs we consider the D-flatness conditions. The D-flatness conditions are given by

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- Using the spectrum given in our model we compute TrQ^A for the three $U_A(1)$s. In a general basis, $Q = \mathrm{diag}[t_1, t_2, t_3, t_4, t_5]$, we have

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- For our model, this trace is computed to be

$$TrQ^A = 61t_1 - 26t_3 + 14t_4 + 11t_5$$

- Applying this to the three $U_A(1)$s leads to

$$\begin{aligned} 5|\theta_{53}|^2 &= 5X \;\; (Q_\chi) \\ -|\theta_{53}|^2 + 4|\theta_{34}|^2 &= 7X \;\; (Q_\psi) \\ 2|\theta_{53}|^2 - 2|\theta_{34}|^2 - 3|\theta_{31}|^2 &= -113X \;\; (Q_\perp) \end{aligned}$$

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- The singlet θ_{14} could get a TeV VEV in order to generate the μ term $\theta_{14} H_u H_d$.

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- Allowing for arbitrary singlet fields to acquire VEVs the dangerous the baryon- and lepton-number violating operators arise through the terms $\theta_{15} L H_u$, $(\theta_{31}\theta_{45} + \theta_{41}\theta_{35}) 10_M \overline{5_3}^2$ and $\theta_{31}\theta_{41} 10_M^3 \overline{5_3}$.

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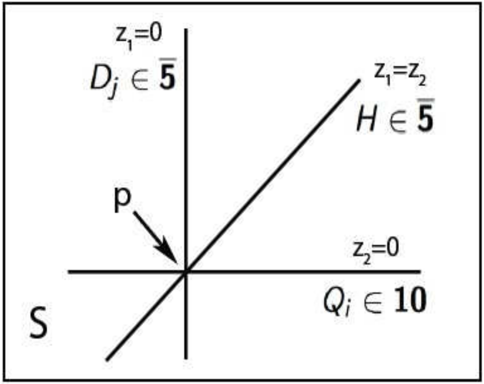
- $\Rightarrow$ provided θ_{15}, θ_{41} and θ_{45} do *not* acquire VEVs these dangerous terms will not arise.

Yukawa Couplings in F-Theory



Yukawa couplings are given by triple wavefunction overlap integrals

- The zero mode wavefunctions are

$$Q_i \propto \left(\frac{z_1}{R_1}\right)^{3-i} e^{-|\lambda_1 z_2|^2}$$

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$$Y_{ij} \propto \int d^2 z_1 d^2 z_2 (\frac{z_1}{R_1})^{3-i} (\frac{z_2}{R_2})^{3-j} \Omega(z_1, \bar{z}_1, z_2, \bar{z}_2)$$

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- As such, we only have a non zero result for the 3,3 component, unless Ω carries non trivial charge under the geometric U(1)s.

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We can use these techniques to understand proton decay operators better

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- Work in progress....