HOW TO REMEDY THE η-PROBLEM OF SUSY GUT HYBRID INFLATION VIA VECTOR BACKREACTION

1 Introduction

- SUSY Hybrid Inflation is undoubtedly one of the most promising types of inflation models that remain into effect today.
- It naturally arises within SUSY GUT models and avoids the massive fine-tuning required in single field inflation models.
- The slowly rolling inflaton stays sub-Planckian during inflation and, thus, non-renormalizable terms are not totally out of control.
- Inflation takes place at energies near the SUSY GUT scale in order to yield the observed magnitude of the curvature perturbation.
- Consequently, the waterfall field can be naturally identified with the GUT Higgs field.
- SUSY is crucial for inflation since it offers a multitude of flat directions along which inflation can take place.
- Moreover, the flatness of these directions is not destroyed by RCs as in non-SUSY theories.
- Actually, in SUSY Hybrid Inflation, the RCs just provide a gentle logarithmic slope needed for the inflaton to slow-roll.
- However, promoting global SUSY to local SUGRA, the need of another set of fine-tunings is introduced.
- Indeed, Kähler corrections to the scalar potential generically give rise to an inflaton mass of order the H.

- This is the infamous η-problem of SUGRA inflation, i.e. the fact that the slow-roll parameter η is pushed to order unity by SUGRA corrections:

$$\eta \equiv m_P^2 \frac{V''}{V} \simeq \frac{1}{3}\left(\frac{m}{H}\right)^2 = \mathcal{O}(1),$$

  with $m \sim H$ being the inflaton mass and prime denoting derivative of the scalar potential V with respect to the inflaton.

- The scalar spectral index n_s of the curvature perturbation then receives a contribution

$$\delta(n_s - 1) = 2\eta = \mathcal{O}(1),$$

  which contradicts its observed approximate scale invariance.

- Moreover, SUGRA corrections lift the flatness of the inflaton direction and destabilize slow-roll.

- As a consequence, not enough e-folds are generated to solve the horizon and flatness problems.

- Finally, if the inflaton mass is $m \gtrsim \frac{3}{2}H$, the inflaton cannot generate the density perturbations.

- Fortunately, for Hybrid Inflation with a minimal Kähler potential, a cancellation protects the model from SUGRA corrections and the η-problem does not arise.

- However, any higher order corrections to the minimal Kähler potential would produce a massive η-problem.

- Recently, a surprising solution to the η-problem was proposed.

- An interaction of the inflaton with a vector boson field leads to a new inflationary attractor solution, where the vector field backreaction $\mathcal{B}_A$ reduces the effective inflaton potential slope: $|V'_{\rm eff}| < |V'|$ with $V'_{\rm eff} \equiv V' + \mathcal{B}_A$.

- This can overcome the η-problem by enabling long-lasting slow-roll to take place even if V is substantially curved.
- Furthermore, the vector backreaction affects the inflaton equation of motion such that it allows the inflaton to undergo particle production even with $\eta = \mathcal{O}(1)$.
- We show that the mechanism of vector backreaction also protects n_s against excessive contributions from a large η parameter.
- We apply these findings to the standard SUGRA Hybrid Inflation model with a generic non minimal Kähler potential.
- We find that Hybrid Inflation can be long-lasting and produce a weakly red spectrum of curvature perturbations in agreement with observations.
- As expected, the contribution of vector fields can give rise to statistical anisotropy in the curvature perturbation ζ.
- The observations still allow as much as 30% statistical anisotropy.
- Moreover, a preferred direction on the microwave sky might be hinted by the unlikely correlation of the low multiples of the CMB.
- One way to generate statistical anisotropy is if the vector field acts as a curvaton, i.e. via the vector field perturbations themselves.
- We apply this mechanism to the standard SUGRA Hybrid Inflation model to see whether it can generate statistical anisotropy in the spectrum and bispectrum of the curvature perturbation.

2 Vector Scaling Slow-Roll Inflation

- Consider a $U(1)$ gauge symmetry with gauge field B_μ and a complex scalar Higgs field Φ with unit charge.

- Writing $\Phi = \phi e^{i\theta}/\sqrt{2}$ and using the gauge invariant combination $h_0 A_\mu \equiv h_0 B_\mu - \partial_\mu \theta$ (h_0=the gauge coupling), we obtain

$$\mathcal{L} = -\frac{1}{4} f(\sigma) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2} h_0^2 \phi^2 A_\mu A^\mu - V_1(\phi),$$

where $F_{\mu\nu}$ is the field strength and $V_1(\phi)$ is the potential for ϕ.

- The gauge kinetic function f is modulated by the real inflaton σ:

$$f(\sigma) \equiv \left(\frac{h_0}{h(\sigma)} \right)^2,$$

with $h(\sigma_0) = h_0$ so that $f(\sigma_0) = 1$ and A_μ becomes canonically normalized when σ assumes its VEV σ_0.

- The spatial components of the physical vector field $W_i = \sqrt{f} A_i / a$ acquire a mass $h_0 \phi_0$ after $U(1)$ breaking (ϕ_0 is the VEV of ϕ).

- As inflation homogenizes A_μ, A_0 vanishes and without loss of generality, we can take $A_\mu = (0, 0, 0, A_z(t))$.

- The metric is $ds^2 = dt^2 - a_1^2(t)(dx^2 + dy^2) - a_2^2(t) dz^2$, where $a_{1,2}$ are the scale factors related to the different spatial directions.

- The average scale factor is $a \equiv (a_1^2 a_2)^{1/3}$ and the average Hubble rate is $H \equiv \dot{a}/a$ (dot denotes derivative w.r.t. cosmic time).

- The anisotropic stress induced by the vector field is then

$$\Sigma \equiv \frac{1}{3H} \frac{d}{dt} \ln \left(\frac{a_1}{a_2} \right).$$

- The coupling of A_μ to σ through the kinetic function $f(\sigma)$ induces a source term $\mathcal{B}_A \equiv -a_2^{-2} f'(\sigma) \dot{A}_z^2 / 2$ in the scalar field equation

$$\ddot{\sigma} + 3H\dot{\sigma} + V'(\sigma) + \mathcal{B}_A(\sigma, \dot{A}_z) = 0,$$

where prime denotes derivative w.r.t. σ, whose potential is $V(\sigma)$.

- For half of the parameter space and for flat $V(\sigma)$, the system evolves to the Standard Slow-Roll (SSR) inflationary attractor.
- On this attractor $\mathcal{B}_A$, Σ, and the vector field energy density ρ_A vanish and, thus, cannot influence the expansion of the Universe.
- For the other half of parameter space, $\mathcal{B}_A$ backreacts on the dynamics of σ and the system tends to the Vector Scaling Slow-Roll (VSSR) inflationary attractor.
- On the VSSR attractor A_μ has a non-negligible effect on the expansion via its non-vanishing ρ_A and Σ.
- On this attractor, $f(\sigma)$ scales as $f_{\text{att}} \propto a^{-4}$, which leads to scale-invariant transverse spectra $\mathcal{P}_{L,R}$ of vector field perturbations.
- From parity conservation, the left and right transverse polarization components $\mathcal{P}_{L,R}$ are identical.
- If A_μ is massless, its longitudinal component decouples and particle production of the vector field is highly anisotropic.
- Thus, A_μ, in this case, can only contribute subdominantly to ζ, but it can still generate substantial statistical anisotropy and anisotropic non-Gaussianity.
- For a massive A_μ, the longitudinal component generates the dominant contribution $\mathcal{P}_\parallel$, which is scale-invariant if $m \propto a$.
- If A_μ becomes heavy by the end of inflation then its spectra become isotropic and can then alone generate ζ.
- The criteria for the existence of the VSSR attractor are

$$\text{Conditions} \quad \begin{cases} \text{I} & |\Gamma_f| \gg 1, \\ \text{II} & |\Gamma_f| \gg |\lambda_0|, \\ \text{III} & \lambda_0 \Gamma_f > 6, \end{cases}$$

where the dimensionless model parameters

$$\Gamma_f(\sigma) \equiv \sqrt{\frac{3}{2}} m_P \left(\frac{f'}{f}\right) \qquad \text{and} \qquad \lambda_0(\sigma) \equiv \sqrt{\frac{3}{2}} m_P \left(\frac{V'}{V}\right)\bigg|_{\phi=0},$$

are used and A_μ is considered massless.

- If the dimensionless model parameters are time-depended, we have additional conditions:

$$A, B, C, D < 1,$$

where

$$\begin{aligned}
A &\equiv 4\sqrt{\frac{2}{3}} m_P \left|\frac{\Gamma_f'}{\Gamma_f^2} + \frac{1}{3}\frac{\lambda_0'}{\Gamma_f^2}\right|, \\
B &\equiv 2\sqrt{\frac{2}{3}} m_P \left|\frac{\lambda_0'}{\Gamma_f^2}\right|, \\
C &\equiv 4\sqrt{\frac{2}{3}} m_P \left|\frac{\Gamma_f'}{\Gamma_f^2} + \frac{\lambda_0'}{\Gamma_f^2} - \frac{\lambda_0'\Gamma_f + \lambda_0\Gamma_f'}{\Gamma_f\left(\lambda_0\Gamma_f - 6\right)}\right|, \\
D &\equiv 4\sqrt{\frac{2}{3}} m_P \left|-\frac{2}{3}\frac{\lambda_0'}{\Gamma_f^2} + \left(\frac{\Gamma_f'}{\Gamma_f^2}\right)\left(\frac{2}{3}\frac{\lambda_0}{\Gamma_f} - \frac{8}{\Gamma_f^2}\right)\right|.
\end{aligned}$$

2.1 General properties of the VSSR attractor

- The vector backreaction $\mathcal{B}_A \equiv -a_2^{-2} f'(\sigma)\dot{A}_z^2/2$ is independent of $\dot{\sigma}$ and thus only modifies the effective slope of the potential $V'_{\text{eff}} \equiv V' + \mathcal{B}_A$.
- Since $f(\sigma) \propto 1/h^2 \to 1$ after the end of inflation, we require that it is always decreasing in time, $\dot{f}(t) < 0$, so that the A_μ remains weakly coupled during inflation.

- Then, since σ decreases in time during inflation, $f'(\sigma) > 0$ and $\mathcal{B}_A < 0$ thereby reducing the effective slope of the potential.
- Once the dimensionless model parameters are slowly varying in time, the backreaction $\mathcal{B}_A$ is proportional to $V'(\sigma)$ and

$$V'_{\rm eff} \equiv V' + \mathcal{B}_A \simeq \frac{6}{\lambda_0 \Gamma_f} V'.$$

- Condition III implies that the effective potential slope seen by the inflaton is reduced.
- So, we can obtain slow-roll inflation with potentials that would normally be too steep for slow-roll.
- Indeed, on the attractor, the slow-roll parameters $\epsilon_{\rm H} \equiv -\dot{H}/H^2$ and $\eta_{\rm H} \equiv -\ddot{H}/2H\dot{H}$ are

$$\epsilon_{\rm H} \simeq \frac{2\lambda_0}{\Gamma_f} \ll 1 \quad \text{and} \quad \eta_{\rm H} \simeq \frac{2\lambda_0}{\Gamma_f} + \frac{\sqrt{6} m_P}{\Gamma_f} \left(\frac{\lambda'_0}{\lambda_0} - \frac{\Gamma'_f}{\Gamma_f} \right).$$

and slow-roll inflation with $\epsilon_{\rm H}, \eta_{\rm H} \ll 1$ is possible.

- From the slow-roll equations

$$3 m_P^2 H^2 \simeq V(\sigma) \qquad \text{and} \qquad 3H\dot{\sigma} \simeq -V'_{\sf eff}(\sigma),$$

we can find the number of e-foldings

$$N_{\sf att} = \frac{1}{4} \ln \frac{f(\sigma_i)}{f(\sigma_{\sf end})},$$

with σ_i and $\sigma_{\sf end}$ the field values at the start and end.

- The vector-to-scalar field energy density ratio $\mathcal{R}$ does not vanish:

$$\mathcal{R} \equiv \frac{\rho_A}{\rho_\sigma} \simeq \frac{\lambda_0 \Gamma_f - 6}{\Gamma_f^2}$$

inducing a small anisotropic stress $\Sigma \simeq 2\mathcal{R}/3$ which can lead to statistical anisotropy in ζ.

3 Curvature perturbation from VSSR inflation

- At horizon crossing of the pivot scale $k_* = 0.002\text{Mpc}^{-1}$, the curvature perturbation generated by inflaton is

$$\frac{2}{5}\zeta_\sigma = \left.\frac{\delta\rho_\sigma}{\rho_\sigma}\right|_* = \left.\frac{1}{5\sqrt{3}\pi}\frac{V^{3/2}}{m_P^3\,|V'_{\rm eff}|}\right|_* \simeq \left.\frac{1}{30\sqrt{2}\pi}\frac{V^{1/2}|\Gamma_f|}{m_P^2}\right|_*,$$

- Considering that the observed $\zeta \simeq 4.8 \times 10^{-5}$ (COBE normalization) is generated by the inflaton alone, we have $\zeta_\sigma \simeq \zeta$.

3.1 The spectral index

- The spectrum of the curvature perturbation, if entirely generated by the inflaton during the VSSR attractor, is

$$\mathcal{P}_\zeta \simeq \left.\frac{1}{4\pi^2}\left(\frac{H^2}{\dot\sigma}\right)^2\right|_* \simeq \left.\frac{1}{24\pi^2 m_P^4}\frac{V(\sigma)}{\epsilon(\sigma)}\left(\frac{\lambda_0\Gamma_f}{6}\right)^2\right|_*,$$

where the slow-roll parameters are defined in the usual way $\epsilon(\sigma) \equiv (m_P^2/2)\,(V'/V)^2$ and $\eta(\sigma) \equiv m_P^2\,(V''/V)$.

- The spectral index $n_s - 1 \equiv \left.\frac{d\ln\mathcal{P}_\zeta}{d\ln k}\right|_*$ on the VSSR attractor is

$$\begin{aligned} n_s - 1 &\simeq \left(\frac{6}{\lambda_0\Gamma_f}\right)\left[2\eta - 6\epsilon - 2m_P\sqrt{\frac{2}{3}}\frac{(\lambda_0\Gamma_f)'}{\Gamma_f}\right], \\ &= -2\left(\frac{6}{\lambda_0\Gamma_f}\right)\left[\epsilon + m_P\sqrt{\frac{2}{3}}\frac{\lambda_0\Gamma'_f}{\Gamma_f}\right]. \end{aligned}$$

- The standard result $n_s - 1 = 2\eta - 6\epsilon$ is recovered for $V'_{\rm eff} = V'$, i.e. $\lambda_0\Gamma_f = 6$, hence $(\lambda_0\Gamma_f)' = 0$.

- We see that n_s is independent of the potential curvature encoded in η and it is easy to obtain a red spectrum favored by the current bound $0.953 \leq n_s \leq 0.981$ (at 1σ).
- The running of the spectral index $n'_s \equiv \frac{dn_s}{d\ln k}$ is also found

$$n'_s \simeq 2\epsilon \left(\frac{6}{\lambda_0 \Gamma_f}\right)^2 \left\{\eta - 2\epsilon + 2m_P^2 \left[\frac{\Gamma''_f}{\Gamma_f} - 2\left(\frac{\Gamma'_f}{\Gamma_f}\right)^2\right]\right\}$$

  and should be compared with the current bound with no gravitational waves: $-0.084 < n'_s < 0.020$ (at 95%cf).

3.2 The tensor spectrum

- The tensor-to-scalar ratio on the VSSR attractor is

$$r \simeq 16\epsilon \left(\frac{6}{\lambda_0 \Gamma_f}\right)^2 = \frac{192}{\Gamma_f^2}$$

  with current bound with no running $r < 0.36$ (at 95%cf).

4 Statistical anisotropy

- Since, during VSSR inflation, the vector field remains massless, but with a small non-zero energy density, it could contribute to ζ.
- After the end of inflation, A_μ becomes heavy with mass M_A and oscillates rapidly behaving like pressureless matter.
- It can then nearly dominate the energy density and imprint its spectra of perturbations.
- The (scale-invariant) spectra of perturbations generated during inflation for a massless A_μ, if $f \propto a^{-4}$, is

$$\mathcal{P}_{L,R} = \mathcal{P}_\sigma = \left(\frac{H_*}{2\pi}\right)^2 \qquad \text{and} \qquad \mathcal{P}_\parallel = 0.$$

- The decay rate Γ_A of the oscillating massive vector field is

$$\Gamma_A = \frac{h_0^2 M_A}{8\pi} = \frac{h_0^3 \phi_0}{8\pi} = H_{\mathsf{dec}},$$

where the subscript 'dec' denotes the epoch of vector field decay.

- For the gravitational effect of A_μ not to be suppressed, its oscillations should last at least a Hubble time before its decay, i.e.

$$\varepsilon \equiv \frac{\Gamma_A}{H_{\mathsf{end}}} \simeq \frac{h_0^3 \phi_0 |\Gamma_f(\sigma_*)|}{32\sqrt{6}\pi^2 m_P \zeta} \lesssim 1,$$

where the subscript 'end' marks the end of inflation.

- From this, we get

$$h_0 \lesssim \left(\frac{32\sqrt{6}\pi^2 m_P \zeta}{\phi_0 |\Gamma_f(\sigma_*)|} \right)^{1/3} .$$

- The anisotropic spectra of perturbations of A_μ generate statistical anisotropy parametrized by g:

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta^{\mathsf{iso}}(k) \left[1 + g \left(\hat{\mathbf{d}} \cdot \hat{\mathbf{k}} \right)^2 + \cdots \right],$$

where $\hat{\mathbf{k}} \equiv \mathbf{k}/k$ and $\hat{\mathbf{d}}$ is the unit vector in the preferred direction.

- The present upper bound on $|g|$ is 0.3, while Planck will reduce it to 0.02.

- Maximal statistical anisotropy is generated if the inflaton decays rapidly after inflation, while the vector curvaton decays later.

- In this optimal case, the statistical anisotropy is found to be

$$|g| \approx \sqrt{\frac{2}{3}} \frac{\phi_0 \mathcal{R}_{\mathsf{end}}}{m_P h_0 \zeta |\Gamma_f(\sigma_*)|}.$$

- For a Planck detectable statistical anisotropy $|g| \gtrsim 0.02$, this gives

$$h_0 \lesssim 50\sqrt{\frac{2}{3}}\frac{\phi_0 \mathcal{R}_{\rm end}}{m_P \zeta |\Gamma_f(\sigma_*)|}.$$

4.1 Anisotropic non-Gaussianity

- The vector curvaton may also generate non-Gaussianity in ζ.
- At present, there is a hint for a non-zero non-linearity parameter $f_{\rm NL}$ characterizing non-Gaussianity: $f_{\rm NL}^{local} = 32 \pm 21$ (at 1σ).
- In the best scenario, the amplitude of non-Gaussianity becomes

$$f_{\rm NL} \simeq \frac{5}{3} g^2 \frac{\sqrt{\varepsilon}}{\mathcal{R}_{\rm end}}.$$

- For Planck detectable non-Gaussianity $f_{\rm NL} \gtrsim \mathcal{O}(1)$, this leads to

$$h_0 \lesssim \frac{25\sqrt{6}}{3888}\frac{\mathcal{R}_{\rm end}^2}{\pi^2 |\Gamma_f^3(\sigma_*)|}\left(\frac{\phi_0}{m_P \zeta}\right)^5.$$

5 Vector Scaling SUGRA Hybrid Inflation

- We now embed the above vector curvaton into a well motivated model of SUSY GUT Hybrid Inflation.
- Consider a simple SUSY GUT model based on the gauge group $G = G_{\rm SM} \times U(1)_{B-L}$ ($G_{\rm SM}$=the Standard Model gauge group), which naturally incorporates standard SUSY Hybrid Inflation.
- G may be thought as part of a larger GUT gauge symmetry.

- The model contains a conjugate pair of $G_{\rm SM}$ singlet superfields Φ and $\bar{\Phi}$ with $B-L=+1$ and -1 breaking $U(1)_{B-L}$ by their VEVs and a gauge singlet S acting as our inflaton.

- Adequate flatness of the inflationary trajectory is guaranteed by a discrete Z_n R-symmetry: $S \to S e^{2\pi i/n}$, $W \to W e^{2\pi i/n}$.

- Note that such symmetries arise in many compactified string theories and can effectively act as continuous.

- The most general W and allowed by the symmetries is

$$W = S \sum_{k_1,k_2=0}^{\infty} A_{k_1 k_2} \left(\Phi\bar{\Phi}\right)^{k_1} \left(S^n\right)^{k_2},$$

  where $A_{k_1 k_2}$ are coefficients with varying dimensions.

- For $n \geq 3$, we can rewrite this as

$$W = \kappa S \left(\Phi\bar{\Phi} - M^2\right) + \text{"non-renormalizable terms"}$$

  with $A_{00} = -\kappa M^2$ and $A_{10} = \kappa$, κ and $M \simeq M_{\rm GUT} > 0$ by field redefinitions, and the non-renormalizable terms m_P suppressed.

- The scalar potential V in SUGRA has the form

$$e^{K/m_P^2} \left[F_{\Phi_i} K^{-1}_{ij*} F_{\Phi^*_j} - 3\frac{|W|^2}{m_P^2} \right] + \frac{1}{2} \sum_{a,b} \left[{\rm Re} f_{ab}(\Phi_i) \right]^{-1} h_a h_b D_a D_b$$

  where K=Kähler potential, f_{ab}=gauge kinetic functions, and

$$K_{ij*} = \frac{\partial^2 K}{\partial \Phi_i \partial \Phi^*_j}, F_{\Phi_i} = \frac{\partial W}{\partial \Phi_i} + \frac{W}{m_P^2} \frac{\partial K}{\partial \Phi_i}, D_a = \Phi_i (T_a)^i_j \frac{\partial K}{\partial \Phi_j} + \xi_a.$$

- Here the subscripts $a, b, \cdots$ label the generators T_a of the gauge group with gauge couplings h_a and ξ_a are Fayet-Iliopoulos D-terms for the $U(1)$ gauge groups.

- In our model, only the gauge kinetic function f for the $U(1)_{B-L}$ is taken not equal to unity and we have $\Phi_i = (S, \Phi, \bar{\Phi})$.
- D-flatness requires that $\bar{\Phi}^* = \Phi e^{i\theta}$ where we choose $\theta = 0$ so that the SUSY vacua are contained.
- Then bringing Φ, $\bar{\Phi}$ on the real axis by a $U(1)_{B-L}$ rotation, we write $\Phi = \bar{\Phi} \equiv \phi/2$ where ϕ is a normalized real scalar field.
- We also define the normalized real scalar field σ: $|\sigma| \equiv \sqrt{2}|S|$ (see below).
- The SUSY minimum is at $\sigma = \sigma_0 = 0$ and $\phi = \phi_0 = \pm 2M$.
- For $|\sigma| > |\sigma_c| = \sqrt{2}M$, V has a stable flat direction at $\phi = 0$ and inflation is driven by the false vacuum energy density $\kappa^2 M^4$.
- As σ crosses its critical value, the effective mass-squared m_ϕ^2 of ϕ

  $$m_\phi^2 \simeq \kappa^2 \left(|S|^2 - M^2\right)$$

  becomes tachyonic and inflation ends abruptly by a waterfall.
- K is a real function of the invariants $|S|^2, |\Phi|^2, |\bar{\Phi}|^2, \Phi\bar{\Phi}$, S^n.
- On the inflationary trajectory, where $\Phi = \bar{\Phi} = 0$, the matrix K_{ij*} becomes diagonal and $F_\Phi = F_{\bar{\Phi}} = 0$.
- So the only terms in K that will contribute on the trajectory are

  $$K = \sum_{k_1,k_2=0}^{\infty} \frac{|S|^{2k_1}}{m_P^{2k_1+nk_2-2}} \left[a_{k_1k_2} \left(S^n\right)^{k_2} + h.c\right],$$

  where $a_{k_1k_2}$ are dimensionless coefficients of order one.
- Hence $K = |S|^2 - (\alpha/4)|S|^4/m_P^2 + \cdots$ with $a_{00} = 0$, $a_{10} = 1/2$ and $a_{20} = -\alpha/8$ where $|\alpha| \sim 1$ is a real parameter.

- The scalar potential in SUGRA on the inflationary trajectory can then be parameterized as

$$V = \kappa^2 M^4 \sum_{k_1,k_2=0}^{\infty} P_{k_1 k_2} \frac{|S|^{2k_1}(S^n)^{k_2}}{m_P^{2k_1+nk_2}} + h.c.,$$

  where the $P_{k_1k_2}$ are dimensionless coefficients that are functions of $A_{k_1k_2}$ and $a_{k_1k_2}$.

- Writing $S = |\sigma|e^{i\vartheta}/\sqrt{2}$, the dimensionless potential $V/\kappa^2 M^4$ is

$$1+\frac{\alpha}{2}\left(\frac{\sigma}{m_P}\right)^2+\beta\left(\frac{\sigma}{m_P}\right)^4+2\gamma(n+1)\left(\frac{|\sigma|}{\sqrt{2}m_P}\right)^n\cos n\vartheta+\cdots$$

  with $8\beta = 1 + 7\alpha/2 + 2\alpha^2 - 18(a_{30} + c.c)$, $\gamma = c + a_{01} - a_{11}$ taken real with c from $A_{01} = -c\kappa M^2/m_P^n$.

- Minimizing V w.r.t ϑ, we find that, for $\gamma < 0$, $\vartheta = 2\pi k/n$, which by a Z_n transformation can be brought to zero.

- For $\gamma > 0$, V is minimized with $\vartheta = (2k+1)\pi/n$, which by a Z_n transformation can be brought to π/n.

- The one-loop RCs to V on the inflationary trajectory are calculated by using the Coleman-Weinberg formula:

$$\Delta V_{1-\text{loop}} = \frac{(\kappa M)^4}{32\pi^2}\left(2\ln\frac{\kappa^2\sigma^2}{2Q^2} + f_c(x)\right),$$

  with $f_c(x) \equiv (x+1)^2\ln(1+1/x) + (x-1)^2\ln(1-1/x)$ where $x \equiv \sigma^2/2M^2$ and Q is a renormalization scale.

- They generate a logarithmic slope on the inflationary valley.

- For sub-Planckian field values $\sigma < m_P$, the scalar fields remain approximately canonically normalized even with a non-minimal K.

- As we see, for non-canonical K, the inflaton obtains a contribution to its mass squared $V''(\phi = 0) \simeq 3\alpha H^2$ leading to $\eta \simeq \alpha$.

- Therefore, σ would normally be fast-rolling unless α is suppressed, which is the infamous η-problem.
- In the VSSR attractor,however, we can obtain slow-roll inflation even without fine-tuning the non-canonical coefficient α.
- In our case, the dimensionless model parameter λ_0 is

$$\sqrt{\frac{3}{2}}\left[\alpha\left(\frac{\sigma}{m_P}\right)+\left(4\beta-\frac{1}{2}\alpha^2\right)\left(\frac{\sigma}{m_P}\right)^3+\frac{\kappa^2}{8\pi^2}\left(\frac{m_P}{\sigma}\right)+\cdots\right].$$

- We assume here natural values for $|\alpha|\sim 1$ so that the first term in λ_0 dominates over the RCs until the end of inflation.
- The condition for this is $\kappa < 4\pi\sqrt{|\alpha|}\left(\frac{M}{m_P}\right)$.
- We will show that a red spectrum can still be obtained, in this case, if the cosmological scales exit during the VSSR attractor.
- We also find, for the first derivative of λ_0, that

$$\sqrt{\frac{2}{3}}m_P\,|\lambda_0'| = |\eta-2\epsilon| \simeq \left|\alpha+3\left(4\beta-\frac{1}{2}\alpha^2\right)\left(\frac{\sigma}{m_P}\right)^2+\cdots\right|.$$

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6 An exponential gauge kinetic function

- The gauge kinetic function, which is holomorphic, cannot contain terms linear in S because of the Z_n symmetry, but combinations S^n are allowed.
- Combinations $\Phi\bar{\Phi}$ are also allowed, but they do not contribute on the inflationary trajectory.

- We consider an exponential gauge kinetic function:

$$f(S^n) = \exp\left[q\left(\frac{S}{M}\right)^n\right] = \exp\left[q\left(\frac{|\sigma|}{\sqrt{2}M}\right)^n e^{in\vartheta}\right],$$

  which goes to unity as the inflaton settles into the SUSY vacuum.

- For $\gamma < 0$, where $\vartheta = 0$, we choose $q > 0$ and for $\gamma > 0$, where $\vartheta = \pi/n$, we choose $q < 0$ so that the exponent is always positive

$$f(\sigma) = e^{|q|\left(|\sigma|/\sqrt{2}M\right)^n}.$$

- The dimensionless model parameter $\Gamma_f(\sigma)$ is then

$$\Gamma_f(\sigma) = |q|n\frac{\sqrt{3}}{2}\left(\frac{m_P}{M}\right)\left(\frac{\sigma}{|\sigma|}\right)\left(\frac{|\sigma|}{\sqrt{2}M}\right)^{n-1}.$$

- We see that it is not constant for $n \neq 1$.
- The VSSR attractor demands that Γ_f and λ_0 have the same sign, so we take the K parameter $\alpha > 0$.
- We also choose $\sigma > 0$ for definiteness.
- For $|q| \sim 1$, conditions I and II for the existence of the VSSR attractor are readily satisfied, while condition III requires that $|q|\alpha n > 4$.
- Since $\lambda_0 = \lambda_0(\sigma)$, $\Gamma_f = \Gamma_f(\sigma)$ are not constants, we have to consider the additional conditions too.
- The tightest ones are $A, C < 1$, which yield

$$|q| \gtrsim \frac{8}{3} - \frac{8}{3n}, \quad \alpha \gtrsim 4\left[n\left(|q| - \frac{8}{3}\right)\right]^{-1},$$

  with the latter being stronger than condition III.

6.1 Properties of vector scaling slow-roll inflation

- The e-foldings N_{att} during the VSSR attractor from an initial inflaton value σ_i until the end of inflation are

$$N_{\text{att}} = \frac{|q|}{4}\left[\left(\frac{\sigma_i}{\sqrt{2}M}\right)^n - 1\right].$$

- For $\sigma_i < m_P$ and, say, $|q| = n = 3$, we find that $N_{\text{att}} \lesssim 10^5$ which is more than enough for solving the horizon and flatness problems.
- At horizon exit of the pivot scale k_*,

$$\frac{\sigma_*}{\sqrt{2}M} = \left(\frac{4N_*}{|q|} + 1\right)^{1/n}.$$

- The reduced effective scalar potential slope is found to be

$$V'_{\text{eff}}/V' \simeq (4/|q|\alpha n)(\sqrt{2}M/\sigma)^n \ll 1.$$

- The slow-roll parameters become

$$\epsilon_{\text{H}} \simeq \frac{2\alpha}{|q|n}\left(\frac{\sigma}{m_P}\right)^2\left(\frac{\sqrt{2}M}{\sigma}\right)^n, \eta_{\text{H}} \simeq \epsilon_{\text{H}} + \frac{2(2-n)}{|q|n}\left(\frac{\sqrt{2}M}{\sigma}\right)^n,$$

and remain $\ll 1$ for $n \geq 3$ and $m_P > \sigma > \sigma_c = \sqrt{2}M$.

- The vector-to-scalar energy density ratio $\mathcal{R}$ increases during VSSR inflation and its value at the end of inflation is

$$\mathcal{R}_{\text{end}} \simeq 2\left(\frac{M}{m_P}\right)^2\left[\frac{|q|\alpha n - 4}{(qn)^2}\right].$$

- With $n = |q| = 3$, $\alpha = 4$, we find that $\mathcal{R}_{\text{end}} \simeq 1 \times 10^{-4}$. For large n, the ratio decreases as $\mathcal{R}_{\text{end}} \propto 1/n$.

6.2 The curvature perturbation and gravitational waves

- The primordial curvature perturbation is found to be

$$\frac{2}{5}\zeta \simeq \frac{\kappa|q|n}{20\sqrt{6}\pi}\left(\frac{M}{m_P}\right)\left(\frac{4N_*}{|q|}+1\right)^{(n-1)/n}.$$

- The COBE normalization with $M = M_{\rm GUT}$, $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$ then leads to $\kappa \lesssim 1.5 \times 10^{-3}$, which readily satisfies the condition for the existence of the VSSR attractor with $\alpha = 4$.

- COBE normalization can be satisfied for $M = M_{\rm GUT}$, whereas, in standard SUSY Hybrid Inflation, M is somewhat below $M_{\rm GUT}$.

- For $|q|, \alpha \sim 1$, the scalar spectral index is, approximately,

$$n_s \simeq 1 - \frac{2(n-1)}{nN_*}.$$

- So, for $n = 3$, $n_s \simeq 0.978$ and, for $n \gg 1$, $n_s \simeq 0.967$, which fit very well within the 1σ bounds from WMAP7.

- The running of n_s is well approximated by

$$n_s' \simeq -\frac{2(n-1)}{nN_*^2}.$$

  which, for $n = 3$, gives $n_s' \simeq -3.6 \times 10^{-4}$ and, for $n \gg 1$, $n_s' \simeq -5.4 \times 10^{-4}$ well satisfying the WMAP7 constraints.

- The tensor-to-scalar ratio is

$$r \simeq \frac{256}{(qn)^2}\left(\frac{M}{m_P}\right)^2\left(\frac{4N_*}{|q|}+1\right)^{2(1-n)/n}.$$

  which, for $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, gives $r \lesssim 1 \times 10^{-6}$, satisfying the WMAP7 bound, but probably too small to be observed.

6.3 Statistical anisotropy and anisotropic non-Gaussianity

- For the gravitational effect of the vector curvaton not to be suppressed $\varepsilon \lesssim 1$, which gives

$$h_0^3 \lesssim \frac{32\sqrt{2}\pi^2\zeta}{|q|n}\left(\frac{4N_*}{|q|}+1\right)^{(1-n)/n},$$

  which, for $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, becomes $h_0 \lesssim 0.05$.

- So, for SUSY GUT $h_0 \sim 0.7$, the gravitational effect of the vector curvaton to be somewhat suppressed.

- For the statistical anisotropy induced in the spectrum

$$|g| \approx \frac{4\sqrt{2}\mathcal{R}_{\rm end}}{3|q|nh_0\zeta}\left(\frac{M}{m_P}\right)^2\left(\frac{4N_*}{|q|}+1\right)^{(1-n)/n}$$

  to be Planck detectable, $h_0 \lesssim 1\times10^{-4}$ for $\alpha \simeq 4$, $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, which is far smaller than the SUSY GUT h_0.

- The amplitude of non-Gaussianity

$$f_{\rm NL} \simeq \frac{5}{3}g^2\frac{\sqrt{\varepsilon}}{\mathcal{R}_{\rm end}},$$

  is detectable, i.e. $f_{\rm NL} \gtrsim \mathcal{O}(1)$, if $h_0 \lesssim 3\times10^{-10}$ for $\alpha \simeq 4$, $N_* = 60$, $n \geq 3$, $|q| \gtrsim 3$, which is far too small.

7 Conclusions

- We showed that, if the inflaton modulates the kinetic function of a vector field, the backreaction to the inflaton's variation allows steep inflation despite sizable Kähler corrections to the potential.

- It also produces a mildly red spectral index of inflaton perturbations eliminating the η-problem of SUGRA inflation.

- We have applied the above to a model of SUGRA Hybrid Inflation, where the waterfall field is taken to be the Higgs field of a GUT.
- The vector field with modulated kinetic function is one of the GUT bosons, which becomes massive at the GUT phase transition.
- We showed that slow-roll inflation can take place with a generic Kähler potential despite the fact that $\eta = \mathcal{O}(1)$.
- Moreover, a red spectrum of perturbations is attained, in agreement with observations.
- Indeed, for an exponential gauge kinetic function, we obtain $n_s \simeq 0.97 - 0.98$ with negligible running and tensor fraction.
- The vector field could contribute to the curvature perturbation ζ if it acts as a vector curvaton.
- We found that the contribution to statistical anisotropy in ζ is important only when the gauge coupling is unnaturally small.
- This is because the attractor solution is such that the vector field contribution to the energy density is rather small.
- So, a long period of vector field oscillations is required after the end of inflation for the vector field to become a significant.
- This requires a small decay width of the vector field and thus a too small gauge coupling.