Knitting neutrino mass textures with or without Tri-Bimaximal mixing

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1 A few known facts

- Fermion mass terms are complex symmetric 3×3 matrices M.

- The Hermitean combination $M^2 = MM^*$can be diagonalised by means of a unitary transformation U to produce real eigenvalues.

  Take U_l and $U_\nu$the corresponding diagonalising matrices for charged leptons and neutrinos respectively. These matrices are by no means unique since

$$U_l \rightarrow U_l C_l$$

$$U_\nu \rightarrow U_\nu D_\nu$$

where

$$C_l = \begin{bmatrix} e^{ic_1} & 0 & 0 \\ 0 & e^{ic_2} & 0 \\ 0 & 0 & e^{ic_3} \end{bmatrix}$$

$$D_l = \begin{bmatrix} e^{id_1} & 0 & 0 \\ 0 & e^{id_2} & 0 \\ 0 & 0 & e^{id_3} \end{bmatrix}$$

are equally good choices.

- Define the lepton mixing matrix as

$$U = U_l^\dagger U_\nu$$

- Parametrise the matrix U using the standard parametrisation

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix}$$

where redundant phases can be removed by suitably choosing the matrices C_l and D_l .

- Experimental evidence shows that

  $\sin^2\theta_{12} \approx 0.312^{+0.019}_{-0.018},\ \sin^2\theta_{23} \approx 0.466^{+0.073}_{-0.058},\ \sin^2\theta_{13} \approx 0.126^{+0.053}_{-0.049}$

  leading remarkably close to the matrix

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{bmatrix}$$

  This is the so called tri-bimaximal (TB) mixing.

2 How to proceed

- Look for mass textures which can reproduce the tri-bimaximal pattern and generalise so that a small θ_{13} angle can be generated.
- Look for symmetric patterns so that the chosen textures can be incorporated into a viable model (possibly superstring inspired).

3 A simple (and effective) solution.

- Take the 3×3 representation of the permutation group S_3.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(123) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (123)^2 = (321)$$

$$(321) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$(12) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (12)^2 = I$$

$$(23) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (23)^2 = I$$

$$(13) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (13)^2 = I \cdot$$

- Use the (alternating) A_3 subgroup of S_3 to construct a hermitean mass matrix for the leptons.

$$M_l^2 = aI + b\,(123) + b^*\,(123)^2 \cdot$$

In terms of eigenvalues,

$$a = \frac{1}{3}\left(m_e^2 + m_\mu^2 + m_\tau^2\right)$$

$$b = \frac{1}{3}\left(m_e^2 + m_\mu^2\omega + m_\tau^2\widetilde{\omega}\right)$$

where $\omega = \exp\left(\frac{2\pi i}{3}\right)$ and $\widetilde{\omega} = \exp\left(-\frac{2\pi i}{3}\right)$.

- The diagonalising matrix of M_l^2 does not depend on the values of the constants (this is an important factor!) and is given by

$$U_l = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\widetilde{\omega}}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\widetilde{\omega}}{\sqrt{3}} \end{bmatrix}.$$

- For tri-bimaximal mixing the mixing matrix is

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{bmatrix}.$$

- The neutrino diagonalising matrix will then be

$$U_\nu = U_l U = \begin{bmatrix} \sqrt{\frac{1}{2}} & 0 & -i\sqrt{\frac{1}{2}} \\ 0 & 1 & 0 \\ \sqrt{\frac{1}{2}} & 0 & i\sqrt{\frac{1}{2}} \end{bmatrix}.$$

Thus, we can generate the symmetric neutrino mass matrix which is

$$M_\nu^2 = \begin{bmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{bmatrix}$$

$$x = \frac{m_1^2 + m_3^2}{2}$$

$$y = \frac{m_1^2 - m_3^2}{2}$$

$$z = m_2^2$$

$$M_\nu^2 = xI + y\,(13) + (z - x - y)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

- In the limit $m_1^2 = m_2^2$ the symmetry breaking term vanishes. This is the Harrison-Perkins-Scott construction (P.L.B 530 (167) 2002).

4 Formulation

- A general Hermitean 3×3 matrix contains 9 independent elements and can be written as

$$M = i \ln U$$

where U a unitary matrix. Using the Cayley-Hamilton theorem we may write

$$M = b_1 I + b_2 U + b_3 U^2$$

where b_1,b_2,b_3 are complex in general.

- Write a Hermitean matrix M in the form

$$M = b_1 I + b_2 U + b_3 U^2$$

where U unitary and $\det U = 1$.The standard CKM form contains four independent elements and has determinant one. Adding six degrees of freedom from the b_i coefficients we have a total of ten, so one D.O.F. is redundant, and can be removed by requiring one eigenvalue of U to be one.

The above expression can be diagonalized by means of a similarity transformation. A diagonal unitary matrix is uniquely defined by

$$U_d = \begin{bmatrix} e^{ia_1} & 0 & 0 \\ 0 & e^{ia_2} & 0 \\ 0 & 0 & e^{ia_3} \end{bmatrix} .$$

One phase can be absorbed into a redefinition of the coefficients b_2 and b_3 while taking the determinant condition into account, we end up with

$$U_d = \begin{bmatrix} e^{ia} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-ia} \end{bmatrix} .$$

(The ordering of the diagonal elements may vary). Denoting by m_1, m_2, m_3 the eigenvalues of M, we have

$$
\begin{aligned}
m_1 &= b_1 + b_2 e^{i\alpha} + b_3 e^{2i\alpha} \\
m_2 &= b_1 + b_2 + b_3 \\
m_3 &= b_1 + b_2 e^{-i\alpha} + b_3 e^{-2i\alpha}
\end{aligned}
$$

or

$$
b_1 = -\frac{1}{4}\left[\exp\left[-\frac{3}{2}i\alpha\right]\ m_1 + \frac{1}{4}\exp\left[\frac{3}{2}i\alpha\right]\ m_3\right]\csc\frac{\alpha}{2}\csc\alpha + \frac{1}{4}\csc^2\frac{\alpha}{2}\, m_2
$$

$$
b_2 = \frac{1}{4}\left[\exp\left[-i\alpha\right](m_1 - m_2) - \exp\left[i\alpha\right](m_2 - m_3)\right]\csc^2\frac{\alpha}{2}
$$

$$
b_3 = -\frac{1}{4}\left[\exp\left[-\frac{i}{2}\alpha\right](m_1 - m_2) - \exp\left[\frac{i}{2}\alpha\right](m_2 - m_3)\right]\csc\frac{\alpha}{2}\csc\alpha\,\cdot
$$

- Assuming the standard parametrisation for the unitary matrix U we have

$$
U = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ -c_{12}c_{23}s_{13}e^{i\delta} + s_{12}s_{23} & -c_{23}s_{12}s_{13}e^{i\delta} - c_{12}s_{23} & c_{13}c_{23} \end{bmatrix}\,\cdot
$$

The requirement to have one eigenvalue equal to one leads to the constraint

$$
\sin\delta\sin\theta_{12}\sin\theta_{13}\sin\theta_{23} = 0\cdot
$$

This condition leads to the following four distinct structures for U.

$$U_1 = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23} & -c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{bmatrix}, \quad \delta = 0$$

$$U_2 = \begin{bmatrix} c_{13} & 0 & e^{-i\delta}s_{13} \\ -s_{13}s_{23}e^{i\delta} & c_{23} & c_{13}s_{23} \\ -c_{23}s_{13}e^{i\delta} & -s_{23} & c_{13}c_{23} \end{bmatrix}, \quad \theta_{12} = 0$$

$$U_3 = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -c_{23}s_{12} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{bmatrix}, \quad \theta_{13} = 0$$

$$U_4 = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -s_{12} & c_{12} & 0 \\ -c_{12}s_{13}e^{i\delta} & -s_{12}s_{13}e^{i\delta} & c_{13} \end{bmatrix}, \quad \theta_{23} = 0\cdot$$

4.1 The Neutrinos

We start with the neutrino sector. Assume that $U_\nu = U_2$.

$$U_\nu = \begin{bmatrix} c_{13} & 0 & e^{-i\delta}s_{13} \\ -s_{13}s_{23}e^{i\delta} & c_{23} & c_{13}s_{23} \\ -c_{23}s_{13}e^{i\delta} & -s_{23} & c_{13}c_{23} \end{bmatrix}$$

By construction, the above matrix admits one eigenvalue equal to one, thus the eigenvalues of U are 1 and $e^{\pm i\alpha}$. The diagonalising matrix for U is difficult to find in simple form, so we introduce the following parametrisation.

$$\tan\theta_{13} = \frac{2z_1}{1-z_1^2}$$

$$\tan\theta_{23} = \frac{2z_1 z_2\sqrt{\left(1+z_1^2\right)\left(1-z_2^2\right)}}{z_2^2 - z_1^2 + 2z_1^2 z_2^2}$$

$$\delta = \theta + \frac{\pi}{2}.$$

Then, the diagonalizing matrix for U is

$$V_\nu\left(z_1, z_2, \theta\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i\theta}\frac{1}{p}\left(z_2 - \imath q^2\, z_1\right) & \sqrt{2}q\imath e^{i\theta} & -e^{i\theta}\frac{1}{p}\left(z_2 + \imath q^2\, z_1\right) \\ \frac{q}{p}\left(z_1 z_2 - \imath\right) & -\sqrt{2}z_2 & \frac{q}{p}\left(z_1 z_2 + \imath\right) \\ p & \sqrt{2}q z_1 & p \end{bmatrix}$$

where p, q are functions of $z_{1,2}$ given by

$$p = \sqrt{\frac{1+z_1^2 z_2^2}{\left(1+z_1^2\right)}}$$

$$q = \sqrt{\frac{1-z_2^2}{1+z_1^2}}\ .$$

We can easily check that

$$V_\nu^\dagger U V_\nu = \begin{bmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\alpha} \end{bmatrix}.$$

The eigenvalue α depends only on z_1, z_2

$$e^{i\alpha} = -\frac{z_1 - iz_2}{z_1 + iz_2},\ \text{or}\ \alpha = \tan^{-1}\frac{z_1}{z_2}.$$

4.2 The Charged Leptons

We choose the ordering of the U matrix eigenvalues to be as follows:

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{-i\alpha} \end{bmatrix} .$$

For the case of charged leptons, we confine ourselves to orthogonal matrices. Therefore, out of the four possible forms we choose

$$U_l = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23} & -c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{bmatrix}$$

An orthogonal matrix can be written as

$$U = e^{i\alpha\widehat{n}\cdot\overrightarrow{s}} = 1 + \sin\alpha\ \widehat{n}\cdot\overrightarrow{s} + (1-\cos\alpha)\ \ (\widehat{n}\cdot\overrightarrow{s})^2$$

where $\hat{n} = (n_1, n_2, n_3)$ is a unit vector and the 3×3 matrices s_i satisfy the conditions

$$[s_i, s_j] = \varepsilon_{ijk} s_k$$

and are explicitly given by

$$s_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$s_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$s_3 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$[s_i, s_j] = \varepsilon_{ijk} s_k.$$

This matrix is diagonalised by means of the matrix

$$V_l\left(n_1, n_2, n_3\right) = \frac{1}{\sqrt{2}\sqrt{n_1^2 + n_3^2}} \begin{bmatrix} \sqrt{2}\sqrt{n_1^2 + n_3^2}n_1 & n_1 n_2 - i n_3 & n_1 n_2 + i n_3 \\ -\sqrt{2}\sqrt{n_1^2 + n_3^2}n_2 & n_1^2 + n_3^2 & n_1^2 + n_3^2 \\ \sqrt{2}\sqrt{n_1^2 + n_3^2}n_3 & n_2 n_3 + i n_1 & n_2 n_3 - i n_1 \end{bmatrix}$$

5 Example: The minimal case

5.1 Neutrinos

Put $z_2 = -1$ to get $\tan\theta_{23} = 0$ whilst for the eigenvalue of the unitary matrix U we get

$$e^{i\alpha} = \frac{i + z_1}{i - z_1}$$

and thus, $\alpha = -\theta_{13}$. This way the U matrix becomes

$$U = \begin{bmatrix} \cos\alpha & 0 & -ie^{i\theta}\sin\alpha \\ 0 & 1 & 0 \\ -ie^{-i\theta}\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

The neutrino mass matrix takes the simple form

$$\begin{bmatrix} \frac{1}{2}\left(m_1^2 + m_3^2\right) & 0 & \frac{1}{2}e^{i\theta}\left(m_3^2 - m_1^2\right) \\ 0 & m_2^2 & 0 \\ \frac{1}{2}e^{-i\theta}\left(m_3^2 - m_1^2\right) & 0 & \frac{1}{2}\left(m_1^2 + m_3^2\right) \end{bmatrix}.$$

For $\theta = \pi$ this is exactly the texture of the neutrino mass matrix introduced in the case of TB mixing.

5.2 Charged leptons

The TB-matrix corresponds to

$$n_1 = \frac{1}{\sqrt{3}}\ ,\ n_2 = -\frac{1}{\sqrt{3}}\ , n_3 = \frac{1}{\sqrt{3}}.$$

5.3 A minimal modification

Put

$$z_2 = -1,\ \theta = \pi + x$$

while we keep the charged lepton diagonalising matrix as above.

The leptonic mixing matrix is given by

$$V_M = e^{\frac{i\pi}{6}} e^{-\frac{ix}{3}} V_\nu^\dagger \left(z_1, -1, x+\pi\right) V_l \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

After some algebra and absorbing the redundant phase factors one finds that

$$V_M = \begin{bmatrix} \sqrt{\frac{2}{3}}\cos\frac{x}{2} & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}}\sin\frac{x}{2} \\ -\sqrt{\frac{2}{3}}\sin\left(\frac{x}{2}+\frac{\pi}{6}\right) & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}}\cos\left(\frac{x}{2}+\frac{\pi}{6}\right) \\ \sqrt{\frac{2}{3}}\sin\left(\frac{x}{2}-\frac{\pi}{6}\right) & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\cos\left(\frac{x}{2}-\frac{\pi}{6}\right) \end{bmatrix}.$$

- The experimental bounds are:

$$\begin{aligned} 0.0871557 < |\sin\theta_{13}| < 0.224931 \\ 0.68728 < |\tan\theta_{12}| < 0.713293 \\ 0.213895 < |\tan\theta_{23}| < 1.09131 \end{aligned}$$

and we have

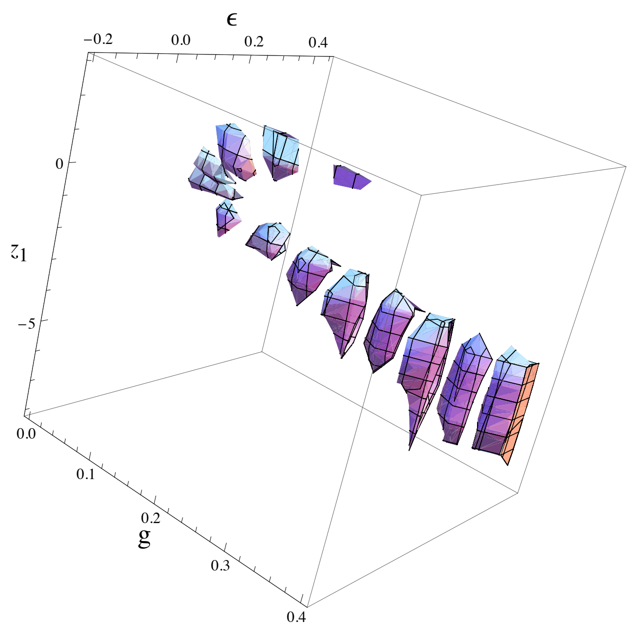
$$
\begin{aligned}
(V_M)_{11} &= \sin\theta_{13} = -\sqrt{\frac{2}{3}}\sin\frac{x}{2} \\
\frac{(V_M)_{23}}{(V_M)_{33}} &= \tan\theta_{23} = -\frac{\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)} \\
\frac{(V_M)_{11}}{(V_M)_{12}} &= \tan\theta_{12} = \frac{1}{\sqrt{2}\cos\frac{x}{2}}.
\end{aligned}
$$

Combining the above, we find that all the constraints are satisfied for

$$\frac{\pi}{15} \lesssim x \lesssim \frac{\pi}{12}.$$

This is a rather interesting result. It shows that a nonzero θ_{13} angle that preserves the symmetric and zero form texture of the M_ℓ^2 and M_ν^2 matrices can be naturally incorporated into the minimal TB-scheme.

- **This is of crucial importance if we really wish to attribute their simple structure to some kind of discrete or other symmetry of the theory .**



6 The general case

We explore now regions of the parameter space which signal departures from the TB-case. Deviations can be easily obtained by assuming for example that

$$n_1 = \frac{1}{\sqrt{3}},\ n_3 = \frac{1}{\sqrt{3}} - \varepsilon,\ n_2 = -\sqrt{1 - n_1^2 - n_3^2}$$

in the charged leptons sector. Similarly, we choose to write

$$z_2 = -1 + g^2$$

in the neutrino diagonalising matrix while we keep $\theta = \pi$.

7 Search for Symmetries

Let us choose $\varepsilon = 1/\sqrt{3}$. This eliminates one entry in V_l which assumes the form

$$V_\ell = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{pmatrix}.$$

Upon inspection, we observe that the remaining two parameters of the neutrino diagonalising matrix can be taken to be

$$z_1 = \tan\frac{7\pi}{12}, z_2 = -\frac{1}{\sqrt{2}}.$$

With this choice, we can readily check that the matrix formed by the moduli of the elements of the leptonic mixing matrix is given by

$$\begin{pmatrix} 0.806056 & 0.586939 & 0.0759986 \\ 0.420639 & 0.655601 & 0.627096 \\ 0.416337 & 0.475067 & 0.775225 \end{pmatrix}$$

pretty much close to the experimental data shown below (the missing elements are determined by unitarity).

$$\begin{bmatrix} \cdots & 0.546431 - 0.580416 & 0.03141 - 0.14091 \\ \cdots & \cdots & 0.63505 - 0.736914 \\ \cdots & \cdots & \cdots \end{bmatrix}.$$

- The charged lepton mass matrix is found to be

$$M_l^2 = \begin{pmatrix} \frac{m_1^2+m_2^2+m_3^2}{3} & -\frac{2m_1^2-m_2^2-m_3^2}{3\sqrt{2}} & -i\frac{m_2^2-m_3^2}{\sqrt{6}} \\ -\frac{2m_1^2-m_2^2-m_3^2}{3\sqrt{2}} & -\frac{4m_1^2+m_2^2+m_3^2}{6} & -i\frac{m_2^2-m_3^2}{2\sqrt{3}} \\ i\frac{m_2^2-m_3^2}{\sqrt{6}} & i\frac{m_2^2-m_3^2}{2\sqrt{3}} & \frac{m_2^2+m_3^2}{2} \end{pmatrix}.$$

This structure can be linked to underlying symmetries. For example, in cases of string derived models with several singlet fields $\phi, \phi', \ldots$ acquiring vevs, one defines expansion parameters $\varepsilon = \langle\phi\rangle/M$, $\varepsilon' = \langle\phi'\rangle/M, \ldots$ with M being the cutoff scale of the higher theory. Then, to leading order $m_3^2 \gg m_2^2 \gg m_1^2$ and we can approximate this matrix by

$$M_l^2 \approx \begin{pmatrix} |\varepsilon|^2 & \varepsilon\bar{\varepsilon}' & \varepsilon \\ \bar{\varepsilon}\varepsilon' & -|\varepsilon'|^2 & \varepsilon' \\ \bar{\varepsilon} & \varepsilon' & 1 \end{pmatrix} m_\ell^2$$

with $\varepsilon = i\sqrt{\frac{2}{3}}, \varepsilon' = \frac{i}{\sqrt{3}}$ and m_ℓ^2 a mass parameter related to charged lepton mass scale.

- The corresponding neutrino mass matrix is given by

$$M_\nu^2 = \begin{pmatrix} \frac{\alpha_+(m_1^2+\xi_- m_2^2+m_3^2)}{4} & \frac{\beta_+(m_1^2-m_3^2)+i\gamma_-(m_1^2-2m_2^2+m_3^2)}{4} & \frac{4\sqrt{2}(m_1^2-m_3^2)-i(m_1^2-2m_2^2+m_3^2)}{16} \\ \cdots & \frac{m_1^2+2m_2^2+m_3^2}{4} & \frac{i\beta_-(m_1^2-m_3^2)+\gamma_+(m_1^2-2m_2^2+m_3^2)}{4} \\ \cdots & \cdots & \frac{\alpha_-(m_1^2+\xi_+ m_2^2+m_3^2)}{4} \end{pmatrix}$$

where the dots stand for the corresponding complex conjugate entries and the various coefficients are

$$\alpha_\pm = \frac{6 \pm \sqrt{3}}{4}, \quad \beta_\pm = \frac{1 \pm \sqrt{3}}{2}, \quad \gamma_\pm = \frac{1}{2}\sqrt{\tan(\pi/4 \pm \pi/6)} = \frac{\sqrt{2 \pm \sqrt{3}}}{2},$$

$$\xi_\pm = \frac{32}{33}\alpha_\pm\gamma_\pm^2.$$

The resulting structure is now more complicated than the corresponding charged lepton one. This is of course to be anticipated in models employing the see-saw mechanism, since the effective neutrino mass matrix is a product of the Dirac and the heavy right handed Majorana neutrino mass matrices $M_\nu \propto m_D M_N^{-1} m_D^T$. Depending on the specific structure of the hypothetical original theory, there are even more options to attribute this matrix to symmetry properties.

8 Conclusions

- We have investigated possible forms for the charged lepton and neutrino mass textures which can reconcile the experimental data on neutrino oscillations.

- This is achieved by using a new parametrisation which disentangles the mass eigenvalues from the corresponding diagonalising matrices.

- We explored the parameter space for allowed deviations from the standard TB scenario and found that the actual data including a non-vanishing θ_{13} angle can be nicely captured, by only introducing a single phase in the $\{13\}$ and $\{31\}$ entries of the neutrino mass texture.

- Neutrino data can still be accommodated even for large deviations from the TB-matrices.