Non-minimal Derivative Couplings in New-minimal N=1 Supergravity

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Classical mechanics

$$L = \frac{1}{2}\dot{x}^2 - V(x)$$

Equation of motion: $\ddot{x} = -V'(x)$

2 time derivatives require 2 initial conditions

$$x(t=0), \dot{x}(t=0)$$

This mean there are 2 canonical coordinates in this theory, Q and P.

$$Energy \propto P^2$$

Ostrogradski instability

Lets consider a generalization

$$L = \frac{1}{2}\dot{x}^2 + \frac{1}{2}\ddot{x}^2 - V(x)$$

Equation of motion: $\ddot{x} - x^{(4)} = -V'(x)$

4 time derivatives require 4 initial conditions, this means there are 4 canonical coordinates.

$$Energy \propto P_1, P_2^2$$

Unstable system!

Dangerous higher derivatives

$$L_{HD} = \frac{1}{2}\phi\Box\phi + \frac{1}{2M^2}\Box\phi\Box\phi - V(\phi)$$

Equation of motion:

$$\Box\phi + \frac{1}{M^2}\Box\Box\phi = -V'(\phi)$$

Time derivatives: $(\partial_t)^4\phi$

Does the instability persist in quantum field theory?

Ghost propagators

$$\frac{1}{2}\phi\Box\phi \longrightarrow \frac{1}{P^2}$$

Massles scalar field

$$\frac{1}{2}\phi\Box\phi + \frac{1}{2M^2}\Box\phi\Box\phi \longrightarrow$$

$$\frac{1}{P^2+\frac{P^4}{M^2}} = \frac{1}{P^2} - \frac{1}{P^2+M^2}$$

Massles scalar and an unphysical state!

Safe higher derivatives?

Yes, known as Galileons!

$$(\partial_\mu \phi)^2 \Box \phi$$

Nicolis,Rattazzi, Trincherini 08'

$$(\partial_\mu \phi)^2 \{ (\Box \phi)^2 - (\partial_\mu \partial_\rho \phi)^2 \}$$

$$(\partial_\mu \phi)^2 \{ (\Box \phi)^3 - 3 \Box \phi (\partial_\mu \partial_\rho \phi)^2 + 2 (\partial_\mu \partial_\rho \phi)^3 \}$$

They can couple to gravity..

Deffayet, Deser, Esposito-Farese 10'-11'

GR-Scalar

$$L_{GRM} = \frac{1}{2\kappa}\sqrt{-g}\,R + \sqrt{-g}\,\frac{1}{2}\phi\Box\phi - \sqrt{-g}\,V(\phi)$$

Propagating degrees off freedom

Massless spin-2 : $h_{\mu\nu}$

Scalar : ϕ

<u>Is this the most general?</u>

Quadratic in matter fields

$$L = L_{GRM} + \frac{1}{M_I^2} L_I + \frac{1}{M_{II}^2} L_{II} + \xi L_{III}$$

Horndeski 74'

$$L_I = \sqrt{-g}\,\phi^2 \left(R_{\mu\nu\kappa\lambda} R^{\mu\nu\kappa\lambda} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right)$$

(Gauss-Bonnet tensor)

$$L_{III} = \sqrt{-g}\,\phi^2 R$$

(Non-minimal non-derivative coupling)

Non-minimal derivative coupling

$$L_{II} = \sqrt{-g}\, G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Kinetic energy of the scalar coupled to gravity through the Einstein tensor

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$$

$$\nabla_\mu G^{\mu\nu} = 0$$ Bianchi identity

Non-minimal derivative coupling

Counting time derivatives in ADM

$$\partial_0 \phi \partial_0 \phi \propto (\partial_t)^2 \qquad \partial_0 \phi \partial_i \phi \propto (\partial_t)$$

$$G^{00} \propto (\partial_t)^2$$ Hamiltonian constraint

$$G^{0\mathrm{i}} \propto (\partial_t)^2$$ Momentum constraint

$$G^{ij} \propto (\partial_{tt})$$ Two time derivatives but not dangerous

Non-minimal derivative coupling

This galileon is the fundamental constituent of the Gravitationally Enhanced Friction mechanism for Inflationary scenarios

Amendola 93'

Kehagias, Germani 10'-11'

Saridakis, Sushkov 10'

This interaction appears in stringy effective field theories

Gross, Sloan 87'

Maeda, Ohta, Wakebe 11'

Supersymmetrize?

$$L_I = \phi^2 (R_{\mu\nu\kappa\lambda} R^{\mu\nu\kappa\lambda} - 4 R_{\mu\nu} R^{\mu\nu} + R^2)$$

$$L_{III} = \phi^2 R$$

Both have been “constructed” in Supergravity.

What about $G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$?

Off-shell N=1 supergravity

Two dominant formulations:

$$e_\mu^a, \psi_\mu, M, b_\nu$$

Old-minimal

Ferrara, Nieuwenhuizen 78'

$$e_\mu^a, \psi_\mu, A_\mu, B_{\mu\nu}$$

New-minimal

Sohnius, West 81'

They differ in the auxiliary sector, which is integrated out, eventually.

New-minimal Supergravity

off-shell gravitational multiplet

$$e_\mu^a, \psi_\mu, A_\mu, B_{\mu\nu}$$

$$\delta A_\mu = -\partial_\mu \phi$$ Gauges the U(1) R-symmetry

$$\delta B_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

The Two-Form

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$$

$$+ \frac{i}{8} \bar{\Psi}_\mu \gamma_\nu \Psi_\lambda + \frac{i}{8} \bar{\Psi}_\nu \gamma_\lambda \Psi_\mu + \frac{i}{8} \bar{\Psi}_\lambda \gamma_\mu \Psi_\nu$$

Super-Covariant Field-Strength

$$H^\mu = -\frac{1}{3!} \epsilon^{\mu\nu\kappa\lambda} H_{\nu\kappa\lambda}$$

Dual of the Field-Strength

$$\hat{D}_a H^a = 0$$

Covariant Derivatives

$$D = d + \delta_L(\omega_{ab}) + \delta_A(A)$$

$$D^{\pm} = d + \delta_L(\omega^{\pm}_{ab}) + \delta_A(A^{\pm})$$

$$\omega^{\pm}_{abc} = \omega_{abc} \pm H_{abc}$$

$$A^{+}_{\mu} = A_{\mu} - H_{\mu}, A^{-}_{\mu} = A_{\mu} - 3\mathrm{H}_{\mu}$$

New-Minimal Supergravity

$$L_{off-shell} = \frac{1}{2} e R + e \bar{\psi}_a r^a + 2 e A_a H^a - 3 e H_a H^a$$

with $r^a = \frac{1}{4} \gamma_5 \gamma_b \epsilon^{bade} \psi_{de}$ and $\psi_{\mu\nu} = D^+_\mu \psi_\nu - D^+_\nu \psi_\mu$

On-Shell $H_\mu = 0$ and $\epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu = 0$

$$L_{sugra} = \frac{1}{2} e R + e \bar{\psi}_a r^a$$

Freedman, van Nieuwenhuizen, Ferrara 76'

Deser, Zumino 76'

General multiplet

$$V = C - \bar{\theta}\chi - \frac{1}{2}\bar{\theta}\{H - i\gamma_5 K + \gamma_5\gamma^a V_a\}\theta$$

$$+ i(\bar{\theta}\theta)\theta\{\gamma_5\lambda + \frac{1}{2}\sigma^a \hat{D}^-_a \chi - \frac{3i\gamma_5}{2}\gamma^a H_a \chi - i\xi C\}$$

$$+ \frac{1}{4}(\bar{\theta}\theta)^2 (D + \frac{1}{2}\hat{\Box}^- C)$$

Characterized by the chiral weight and the spin of its lowest component.

This is a reducible representation of Supersymmetry.

To find the irreducible one can impose constraints.

Chiral multiplet - Chiral projection

$$\nabla_{\dot{\alpha}} \Phi = 0$$

$$\Phi = (A, \chi, F)$$

Chiral weight adjustable

$$\Pi(V) = -\frac{1}{4}\bar{\nabla}^2 V$$

If V has chiral weight n, its chiral projection has n+1

Einstein Multiplet

$$E_a = E_a^*, \quad \nabla^2 E_a = \bar{\nabla}^2 E_a = 0$$

$$\nabla^a E_a = 0$$ Bianchi identity

$$E_a = (H_a, i\gamma_5 r_a, \frac{1}{2}(\hat{G}^+_{ab} - {}^*\hat{F}^+_{ab}))$$

Ferrara, Sabharwal 88'

It is a liner multiplet and it has fixed vanishing chiral weight

Kinetic term n=0

$$L_{kin} = \int d^2\theta \, E \, \Phi \left[-\frac{1}{4} \bar{\nabla}^2 \Phi^\dagger \right] + h.c.$$

It is by construction R-invariant

For a chiral superfield with n=0, the super-integration gives (keeping only bosonic fields)

$$L_{kin} = 2\mathrm{e} \mathrm{A} \Box A^* + 2 e F F^*$$
$$- 2\mathrm{ieH}^c (A \partial_c A^* - A^* \partial_c A)$$

Superspace non-minimal derivative coupling

$$L_{int} = \int d^2\theta\, E\{-\frac{i}{4}\bar{\nabla}^2[\Phi^\dagger E^a \bar{\nabla}_a \Phi]\} + h.c.$$

For a chiral superfield with n=0, the super-integration gives

$$L_{int} = e\, G^{ab}\partial_a A \partial_b A^* + 2\, e\, F\, F^* H^a A_a - 2\, e\, F\, F^* H^a H_a$$

$$+ ieH^a(F^*\partial_a F - F\partial_a F^*) - e\,\partial_b A \partial^b A^* H^a_{\ aH}$$

$$+ 2\, e\, H^a \partial_a A\, H^b \partial_b A^* - ieH_c(\partial_b A^* D^c \partial^b A - \partial_b A\, D^c \partial^b A^*)$$

Full theory in components

$$L_{tot} = \frac{1}{\kappa^2} L_{off-shell} + \frac{1}{2} L_{kin} + \omega^2 L_{int}$$

$$= \frac{1}{\kappa^2} [\frac{1}{2} e R + 2 e A_a H^a - 3 e H_a H^a]$$

$$+ eA \Box A^* + eFF^* - ieH^c (A \partial_c A^* - A^* \partial_c A)$$

$$+ \omega^2 \{ e G^{ab} \partial_a A \partial_b A^* + 2 e F F^* H^a A_a - 2 e F F^* H^a H_a$$

$$+ ieH^a (F^* \partial_a F - F \partial_a F^*) - e \partial_b A \partial^b A^* H_a H^a$$

$$+ 2 e H^a \partial_a A H^b \partial_b A^* - ieH_c (\partial_b A^* D^c \partial^b A - \partial_b A D^c \partial^b A^*) \}$$

Elimination of auxiliaries

From the equations of motion

$$H_\mu = 0 \qquad F = F^* = 0$$

$$\epsilon^{\mu\nu\rho\sigma} \partial_\mu V_\nu = 0$$

Redefined auxiliary

On-Shell

$$L_{tot} = \frac{1}{2\kappa^2} e R + e A \Box A^* + \omega^2 e G^{ab} \partial_a A \partial_b A^*$$

F.F, Germani, Kehagias, Saridakis 12'

Generalization

One can generalize to an arbitrary, holomorphic function of the chiral superfield

$$\int d^2\theta\, E\left\{-\frac{i}{4}\bar{\nabla}^2\left[\bar{W}(\Phi^\dagger)\,E^a\,\nabla_a^{-} W(\Phi)\right]\right\}+h.c.$$

After super-integration and elimination of auxiliaries

$$L_{tot}=\frac{1}{2\kappa^2}e\,R+eA\,\Box\,A^*$$

$$+\omega^2\left|\frac{\partial W}{\partial A}\right|^2 e\,G^{ab}\,\partial_a A\,\partial_b A^*$$

Open Questions

Potential, n different than zero?

Breaking SUSY?

Interaction with Gauge Fields?

Other Safe Higher Derivative Terms?

Old-minimal Supergravity?