Strongly Coupled Anisotropic Plasma in AdS/CFT

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Outline

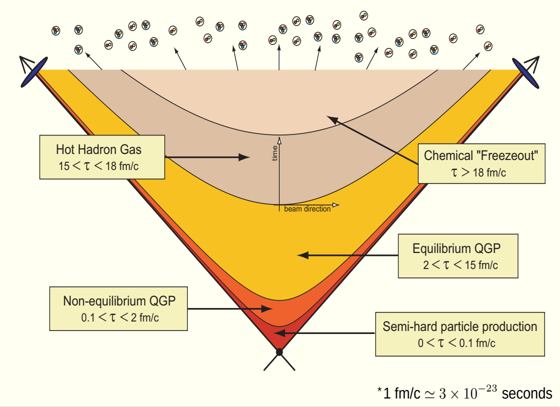
AdS/CFT correspondence

- The AdS/CFT correspondence, in the original and best understood form, is a duality between the $\mathcal{N} = 4$ supersymmetric Yang-Mills and type IIB superstring theory on $AdS_5 \times S^5$.
- In this correspondence there exist a map between gauge invariant operators in field theory and states in string theory.
- Example:The Wilson loop, is a physical gauge invariant object and can measure the interaction potential between the external quarks and acts as an order of confinement.
- The Wilson loop operator in the fundamental representation is dual to a string worldsheet extending in the $AdS_5 \times S^5$ with boundary the actual loop placed on the AdS boundary. *[Maldacena; Rey, Yee, 1998]*

$$< W[C] >= e^{-S_{string[C]}}$$

Why do we need anisotropic gauge/gravity dualities?

- It is interesting theoretically to study thermodynamics and observables in anisotropic IIB SUGRA solutions. The modification of the results due anisotropy and their dependence on it, can be found.
- In early stages the QGP is not in equilibrium. It appears to have anisotropies.



(pic from M. Strickland)

Longitudinal expansion

- By considering completely central collisions we focus more on the longitudinal anisotropic expansion.
- At $\tau = \tau_0$ the partonic momentum distributions can be assumed isotropic. Rapid longitudinal expansion along the beam line happens. Local pressure and momentum anisotropy $P_L < P_T$ occurs.
- Finally the plasma becomes and remains isotropic for $\tau \geq \tau_{iso}$. After this time hydrodynamics can be applied.
- Lots of work in gauge/gravity duality for $\tau \geq \tau_{iso}$. Here we study certain quantities in the earlier time where the plasma is anisotropic.

Motivation

- The rapid expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Properties of the supergravity solutions, that are dual to the anisotropic plasmas.
- There exist several results for observables in weakly coupled anisotropic plasmas. Do their predictions carry on in the strongly coupled limit models?
- The main question we answer accurately here is: How the inclusion of anisotropy modifies the results on several observables in our dual QGP compared to the isotropic theory?

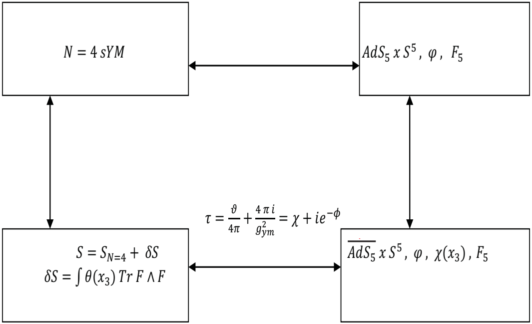
Introduction of Anisotropy

- Introduction of additional D7 branes which create the anisotropy.

[Azeyanagi, Li, Takayanagi, 2009]

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Gauge/Gravity deformation diagram.



The anisotropic background

The metric in string frame [Mateos, Trancanelli, 2011]

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}\mathcal{B}\,dx_0^2 + dx_1^2 + dx_2^2 + \mathcal{H}d{x_3}^2 + \frac{du^2}{\mathcal{F}}\right) + \mathcal{Z}\,d\Omega_{S^5}^2\,.$$

The functions $\mathcal{F}, \mathcal{B}, \mathcal{H}$ depend on the radial direction u and the anisotropy α. In sufficiently high temperatures, $T \gg \alpha$, and imposed boundary conditions the Einstein equations can be solved analytically:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24 u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}$$

• The limit $\alpha \to 0$ reproduce the well known result of the isotropic D3-brane solution (dual to finite T $\mathcal{N} = 4$ sYM solution).

The metric can be expressed in α, T parameters through

$$u_h = \frac{1}{\pi T} + \alpha^2 \frac{5 log 2 - 2}{48\pi^3 T^3} .$$

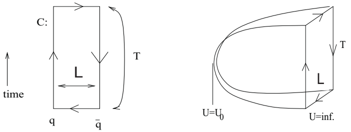
The energy and pressures can be found from the expectation value of the stress tensor, where the elements

$$\begin{aligned} P_\perp := P_{x_1 x_2} &= \frac{\pi^2 N_c^2 T^4}{8} + \alpha^2 \frac{N_c^2 T^2}{32}. \\ P_\parallel := P_{x_3} &= \frac{\pi^2 N_c^2 T^4}{8} - \alpha^2 \frac{N_c^2 T^2}{32}. \end{aligned}$$

$$P_{x_3} < P_{x_1 x_2}$$

Static Potential

The static potential can be measured by introducing two infinitely heavy probe quarks on the boundary of the space. This corresponds to a Wilson loop of the following shape:



(pic taken from 0712.0689)

The normalized expectation value of the Wilson loop which involves the minimal surface of the particular world-sheet minus the infinite quark mass is

$$W[C] \sim e^{-(S - mass_Q)} \sim e^{-V_{Q\bar{Q}} T}$$

Static Potential in the anisotropic background

- Instead of carrying the exact anisotropic metric we rename its elements to the generic metric:

$$ds^2 = g_{00} dx_0^2 + \sum g_{ii} dx_i^2 + g_{uu} du^2 + \text{internal space}$$

u:radial direction, x_i:space-time coordinates

- We align the $Q\bar{Q}$ pair along the anisotropic direction and then along the transverse direction.
- We consider a string world-sheet (τ, σ) of the following form.

$$x_0 = \tau, \qquad x_p = \sigma, \qquad u = u(\sigma) \; .$$

The x_p is the direction where the pair is aligned:
$x_p = x_2 =: x_\perp$ pair along transverse direction,
$x_p = x_3 =: x_\parallel$ pair along parallel direction to anisotropy.

The solution to Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\tilde{g}}$$

is a catenary shape w-s with u_0 being the turning point.

To find the static potential we need to derive from the eoms of the NG action the length L of the Wilson loop. Then express the minimal surface ($\sim$static potential) in terms of L. The process is not always doable analytically. In general the length of the two endpoints of the string on the boundary is given by

$$L = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu}c_0^2}{(g_{00}g_{pp}+c_0^2)g_{pp}}} \, .$$

Which should be inverted as $u_0(L)$. The normalized energy of the string is

$$2\pi\alpha' V \quad = \quad c_0 L + 2\left[\int_{u_0}^{\infty} du\sqrt{-g_{uu}g_{00}}\left(\sqrt{1+\frac{c_0^2}{g_{pp}g_{00}}}-1\right) - \int_{u_h}^{u_0} du\sqrt{-g_{00}g_{uu}}\right] \, .$$

[Sonnenschein,..]

Therefore we can always at least numerically find the $V(L)$ expression for any background. In the anisotropic case we get:

• $V_{\parallel} < V_{\perp} < V_{iso}$ when the comparison is done with LT keeping α, T fixed.

• $\alpha_1 < \alpha_2 \Rightarrow V_{\parallel_1} > V_{\parallel_2}$. Increase of anisotropy, leads to decrease of the static potential.

• The critical length of the string beyond the quarks are not bounded is decreased in presence of anisotropy as $L_{c\parallel} < L_{c\perp} < L_{c\ iso}$.

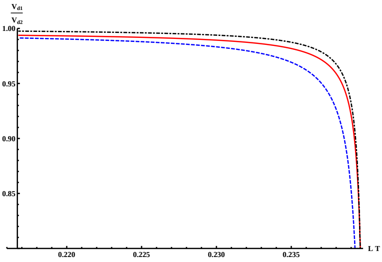


Figure: $V_{\parallel}/V_{\perp}$, $V_{\parallel}/V_{iso}$, $V_{\perp}/V_{iso}$ vs LT and $T = 3$, $\alpha = 0.35T$.

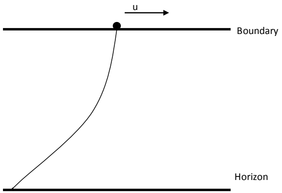
Drag Force

In AdS/CFT the drag force of a single quark moving in the anisotropic plasma can be represented by a trailing string from the boundary where the probe quark moves with the constant speed, to the horizon of the black hole. [Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser, 2006]

String Configuration

In radial gauge the trailing string motion along the $x_p := x_{\parallel,\perp}$ directions described by:

$$x_0 = \tau, \qquad u = \sigma, \qquad x_p = v\tau + \xi(u)$$



By solving the Nambu-Goto equations and after some algebra the drag force can be found for any background to be

$$F_d = -\Pi_u^1 = -\sqrt{\lambda}\frac{\sqrt{-g_{00}g_{pp}}}{(2\pi)}\Big|_{u=u_0}$$

where here u_0 is given by

$$\left(g_{uu}(g_{00} + g_{pp}v^2)\right)\,|_{u=u_0} = 0\ .$$

We calculate the drag force for a quark moving along the anisotropic direction and then the transverse one.

Drag Force results

The qualitative behavior is

- $F_{\parallel} > F_{iso}$
- $F_{\perp} > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_{\perp} < F_{iso}$.
- $$\frac{F_{\parallel}}{F_{\perp}} = 1 + \alpha^2 \frac{(2 - v^2)\,\mathrm{Log}\left[1 + \sqrt{1 - v^2}\right]}{8\pi^2 T^2 (1 - v^2)} \ .$$

For any velocity: $F_{drag,\parallel} > F_{drag,\perp}$

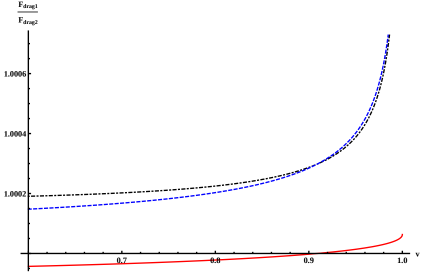


Figure: $F_{drag,\parallel}/F_{drag,\perp}$, $F_{drag,\parallel}/F_{drag,iso}$, $F_{drag,\perp}/F_{drag,iso}$, vs v, $\alpha = 0.1$ and $T = 1$.

Diffusion time

Therefore the diffusion time τ_D is given by:

$$\tau_{D,\|,\perp} = \frac{1}{n_{D,\|,\perp}} = -\frac{1}{F_{drag,\|,\perp}} \frac{M_q v}{\sqrt{1-v^2}} \ ,$$

Remark

The quark mass receives corrections from the thermal medium. The corrected mass does not depend on the directions along the probe quark moves in the anisotropic background but depend on the anisotropic parameter.

The diffusion times along the different directions are:

$$\frac{\tau_{D,\|}}{\tau_{D,\perp}} = 1 - \alpha^2 \frac{(2-v^2)\,\mathrm{Log}\left[1+\sqrt{1-v^2}\right]}{8\pi^2 T^2 (1-v^2)}$$

And if we consider sub-leading the thermal mass corrections then the relations between the diffusion times in different directions are inverse to the drag force ones.

The jet Quenching

In the gravity dual description the jet quenching can be calculated from a minimal surface which ends on an orthogonal Wilson loop lying along the light-like lines where $L_- \gg L_\perp$.

$$\langle W(\mathcal{C}) \rangle = \exp^{-\frac{1}{4\sqrt{2}}\hat{q} L_\perp^2 L_-}$$

[Liu,Rajagopal,Wiedermann,2006]

In isotropic theory we have one jet quenching parameter, in the anisotropic three.

All the possible inequivalent combinations give:

$\hat{q}$	x_p	x_k	Energetic parton along	Momentum broadening along
$\hat{q}_{\perp(\parallel)}$	$x_\perp$	$x_\parallel$	$x_\perp$	$x_\parallel$
$\hat{q}_{\parallel(\perp)}$	$x_\parallel$	$x_\perp$	$x_\parallel$	$x_\perp$
$\hat{q}_{\perp(\perp)}$	$x_{\perp,1}$	$x_{\perp,2}$	$x_{\perp,1}$	$x_{\perp,2}$

where under approximations the generic jet quenching is given by

$$\hat{q}_{p\ (k)} = \frac{\sqrt{2}}{\pi\alpha'}\left(\int_0^{u_h}\frac{1}{g_{kk}}\sqrt{\frac{g_{uu}}{g_{--}}}\right)^{-1}.$$

- The final result is:

$\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.

$\hat{q}$(q motion parallel to anisotropy, broadening along transverse)> $\hat{q}$(q motion transverse to anisotropy, broadening along parallel)> $\hat{q}$(q motion transverse to anisotropy, broadening along transverse)

Other Extensions

- Extensions of the jet quenching and the drag force calculations to generic directions and larger anisotropies have been done. [Chernicoff, Fernandez, Mateos, Trancanelli, 2012a,b].
- Results get different for larger anisotropies but also the inequality of pressures does differ.

Anisotropic momentum distribution function

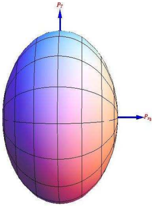
The anisotropic distribution function that can be written as

$$f_{aniso} = c_{norm}(\xi) f_{iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

where [Romatschke, Strickland, 2003]

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$

and **n** the unit vector along the anisotropic direction.



The anisotropic parameter can be related with:

$$T \gg \alpha \Rightarrow \xi \ll 1 \Rightarrow \xi \backsimeq \frac{5\alpha^2}{8\pi^2 T^2} \ .$$

Conclusions-Partial List of Results

We have calculated several observables in a IIB supergravity solution dual to an anisotropic finite temperature $\mathcal{N} = 4$ sYM plasma.

- The static potential:
  - $V_{\parallel} < V_{\perp} < V_{iso}$.
  - $\alpha_1 < \alpha_2 \Rightarrow V_{\parallel_1} > V_{\parallel_2}$.
- The drag Force:
  - $F_{\parallel} > F_{iso}$ and $F_{\parallel} > F_{\perp}$.
  - $F_{\perp} > F_{iso}$ for $v > v_c \simeq 0.9$, while below this velocity $F_{\perp} < F_{iso}$.
- The jet quenching:
  - $\hat{q}_{\parallel(\perp)} > \hat{q}_{\perp(\parallel)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.
- Qualitative "comparison" and partial agreement with weakly coupled results and some data.

Additional information

Modifications with inclusion of dynamical quarks

- Inclusion of dynamical quarks in isotropic theories leads to screening of the static potential. [Satz; Karsch, Kharzeev, Satz; D.G., Irges, ..]
- Inclusion of anisotropy in presence of dynamical quarks does not certainly means further screening. For example in case that the density of dynamical quarks depends on the anisotropy, screening or strengthening may happen.
- Weak coupling models have found increased potential in presence anisotropy in the order $V_{||} > V_{\perp} > V_{iso}$. Inverse order with our strongly coupled results but still the anisotropic direction is affected mostly. Many differences however between the models. [Dumitru, Guo, Strickland, 2007]

To calculate the corresponding Wilson loop we go to the light-cone coordinates as $\sqrt{2}x^{\pm} = x_0 \pm x_p$ where $i, p, k = 1, 2, 3$ A generic metric becomes

$$ds^2 = g_{--}(dx_+^2 + dx_-^2) + g_{+-}(dx_+ dx_-) + g_{ii(i \neq p)} dx_i^2 + g_{uu} du^2$$

$$g_{--} = \frac{1}{2}(g_{00} + g_{pp}), \qquad g_{+-} = g_{00} - g_{pp}$$

The ansatz that describes the string configuration and solves the eom is

$$x_- = \tau, \quad x_k = \sigma, \quad u = u(\sigma)$$

$$x_+, \ x_{p \neq k} \quad \text{are constant },$$

which represents a Wilson loop extending along the x_k direction and lying at a constant $x_+, x_{i \neq k}$. The index k here denotes a chosen direction.

$\hat{q}$	x_p	x_k	Energetic parton along	Momentum broadening along
$\hat{q}_{\perp(\parallel)}$	$x_\perp$	$x_\parallel$	$x_\perp$	$x_\parallel$
$\hat{q}_{\parallel(\perp)}$	$x_\parallel$	$x_\perp$	$x_\parallel$	$x_\perp$
$\hat{q}_{\perp(\perp)}$	$x_{\perp,1}$	$x_{\perp,2}$	$x_{\perp,1}$	$x_{\perp,2}$

After calculating the on-shell action, canceling the divergences and applying the approximations we obtain

$$\hat{q}_{p\ (k)} = \frac{\sqrt{2}}{\pi\alpha'}\left(\int_0^{u_h}\frac{1}{g_{kk}}\sqrt{\frac{g_{uu}}{g_{--}}}\right)^{-1}.$$

Applying the results to our background we obtain:

- $\hat{q}_{\|(\perp)} > \hat{q}_{\perp(\|)} > \hat{q}_{\perp(\perp)} > \hat{q}_{iso}$.

$\hat{q}$(q motion parallel to anisotropy, broadening along transverse)$>$ $\hat{q}$(q motion transverse to anisotropy, broadening along parallel)$>$ $\hat{q}$(q motion transverse to anisotropy, broadening along transverse) • $\frac{\hat{q}_{\|(\perp)}}{\hat{q}_{iso}} \simeq 1 + 0.122\frac{\alpha}{T}$

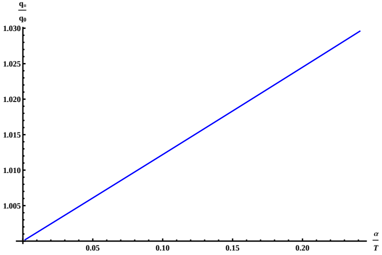


Figure: $\hat{q}_{\|(\perp)}/\hat{q}_{iso}$ vs α/T. $T = 5$.

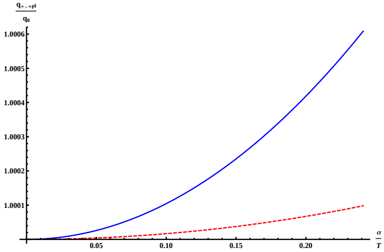


Figure: $\hat{q}_{\perp}$, $\hat{q}_{\perp(\perp)}$ vs α/T. $T = 5$.

To relate ξ and α define

$$\Delta := \frac{P_T}{P_L} - 1 = \frac{P_{x_1 x_2}}{P_{x_3}} - 1 \ .$$

Using the anisotropic distribution function: [Martinez, Strickland, 2009]

$$\Delta = \frac{1}{2}(\xi - 3) + \xi \left((1+\xi) \frac{\arctan \sqrt{\xi}}{\sqrt{\xi}} - 1 \right)^{-1}$$

Using the supergravity model

$$\Delta = \frac{\alpha^2}{2\pi^2 T^2} \ .$$

For

$$T \gg \alpha \Rightarrow \xi \ll 1 \Rightarrow \xi \backsimeq \frac{5\alpha^2}{8\pi^2 T^2} \ .$$

Supposing we trust the estimation of the anisotropic parameter $\xi \simeq 1$ obtained from

$$\xi = \frac{10\eta}{T\tau s} \ .$$

and using any comparison normalization scheme(direct or fixed energy density scheme)

$$\xi_{aSYM} \gtrsim \xi \ .$$

Therefore, in that case we can not make a more ‘quantitative’ comparison using our model in the particular limit $T \gg \alpha$. We can do it only if the values of $\xi \ll 1$.
But we have found the qualitative behavior on how the observables behave in the strong coupling in presence of anisotropy.

$$P(k_\perp) = \int d^2x_\perp \, e^{-ik_\perp \cdot x_\perp} \, \mathcal{W}_\mathcal{R}(x_\perp)$$

$$\mathcal{W}_\mathcal{R}(x_\perp) = \frac{1}{d\,(\mathcal{R})} \left\langle \mathrm{Tr} \left[W^\dagger_\mathcal{R}[0, x_\perp] \, W_\mathcal{R}[0,0] \right] \right\rangle$$

$$W_\mathcal{R}\left[x^+, x_\perp\right] \equiv P \left\{ \exp \left[ig \int_0^{L^-} dx^- \, A^+_\mathcal{R}(x^+, x^-, x_\perp) \right] \right\}$$

$$\hat{q} \equiv \frac{\langle k^2_\perp \rangle}{L} = \frac{1}{L} \int \frac{d^2k_\perp}{(2\pi)^2} k^2_\perp P(k_\perp)$$